INSTITUTE
FOR
AEROSPACE STUDIES

UNIVERSITY OF TORONTO

AFOSR - TR - 72 - 1278

NEAR-FREE-MOLECULE FLOW CALCULATIONS-

MONTE CARLO STUDY OF SKIMMER INTERACTIONS

by

N.A. Derzko

A

October, 1971.   UTIAS Technical Note No. 169

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

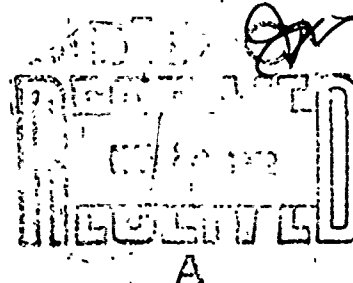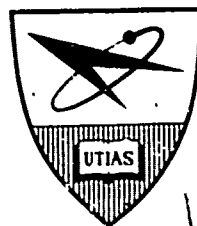| 1. ORIGINATING ACTIVITY *(Corporate author)*	2a. REPORT SECURITY CLASSIFICATION
UNIVERSITY OF TORONTO INSTITUTE FOR AEROSPACE STUDIES TORONTO 5, ONTARIO, CANADA	UNCLASSIFIED
	2b. GROUP

3. REPORT TITLE

NEAR-FREE-MOLECULE FLOW CALCULATIONS-MONTE CARLO STUDY OF SKIMMER INTERACTIONS

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*
Scientific   Interim

5. AUTHOR(S) *(First name, middle initial, last name)*

N A DERZKO

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
October 1971	11	8
8a. CONTRACT OR GRANT NO. AFOSR-1481-68	9a. ORIGINATOR'S REPORT NUMBER(S)
b. PROJECT NO. 9783-01	UTIAS T.N. NO. 169
c. 61102F 681307	9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)* AFOSR - TR - 72 - 1 2 7 8

10. DISTRIBUTION STATEMENT

Approved for public release; distribution unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
TECH, OTHER	AF Office of Scientific Research (NAM) 1400 Wilson Boulevard Arlington, Virginia 22209

13. ABSTRACT

The Monte Carlo simulation method of G. A. Bird was applied to the study of flow through a skimmer using various geometries, speed ratios, and Knudsen numbers. The computer program followed a sample of 3500 hard sphere molecules, accumulating distribution function profiles in each of 25 regions of the flowfield, the beam profile at the downstream end of the flowfield, and the mass flow through the skimmer orifice. The results show beam intensities of the order of .3 of the no skimmer interaction value at Knudsen numbers of 01 and demonstrate clearly that most of the interaction occurs from 0 to 4 mean free paths upstream of the orifice.

DD FORM 1473
NOV 65

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
SKIMMER INTERACTION						
MONTE CARLO SIMULATION						
BOLTZMANN EQUATION						
MOLECULAR BEAM						

NEAR-FREE-MOLECULE FLOW CALCULATIONS-
MONTE CARLO STUDY OF SKIMMER INTERACTIONS

by

N. A. Derzko

Submitted March, 1971.

October, 1971.

UTIAS Technical Note No. 169

ACKNOWLEDGEMENTS

SUMMARY

The Monte Carlo simulation method of G. A. Bird was applied to the study of flow through a skimmer using various geometries, speed ratios, and Knudsen numbers. The computer program followed a sample of 3500 hard sphere molecules, accumulating distribution function profiles in each of 25 regions of the flowfield, the beam profile at the downstream end of the flowfield, and the mass flow through the skimmer orifice.

The results show beam intensities of the order of .3 of the no skimmer interaction value at Knudsen numbers of 01 and demonstrate clearly that most of the interaction occurs from 0 to 4 mean free paths upstream of the orifice.

TABLE OF CONTENTS

INTRODUCTION

This report deals with part of the work we have been doing during the past three summers on the development and testing of a number of computer programs designed to simulate molecular flow problems in the transition regime. The simulation is done with hard sphere model using the Monte Carlo method developed by G. A. Bird (2).

We describe here its application to the study of the flow through a skimmer under conditions similar to those involved in the production of molecular beams of high intensity and speed ratio. We simulate the flow which occurs in a cylindrical region reaching upstream and downstream of the skimmer but not including the nozzle.

Since its proposal by Kantrowitz and Grey in 1951, the free jet-skimmer combination has proved far superior to the older methods using the effusive flow from an orifice in an oven source. Nevertheless, in practice the intensity of a beam produced in this way is substantially lower than that one would obtain if the skimmer just removed the portion of the free jet around the beam region - that is, did a perfect job. Considerable experimental work has gone into studying the nature and causes of this effect. Recently Govers, LeRoy and Deckers (6) have studied the effects of background scattering on skimmer interaction; Bossel, Hurlbut, and Sherman (3) have done experiments to study the magnitude of the skimmer interaction for various geometries and jets.

The effect of the skimmer interaction, whatever its nature, is quite clear If one plots beam intensity in particles per sec. per steradian vs nozzle skimmer separation, the graph starts at a low value for small separation, increases to a maximum and then drops rapidly as the separation exceeds the distance to the Mach disc (Fig. 4, p. 994, 6). More or less successful attempts have been made to correlate the large amount of experimental data available to enable one to describe a given situation with a fairly small number of parameters With skimmers of similar geometry, the ratio of mean free path to skimmer orifice diameter appears to be significant Roger Campargue (4) has discussed a number of dimensionless quantities relevant to the nozzle beam apparatus.

The theory associated with a nozzle beam divides naturally into three areas:

(a) Description of the free jet - that is the density, velocity, and speed ratio at every point downstream of the nozzle.

(b) Skimmer interaction.

(c) Development of the beam downstream of the skimmer (assuming that any molecules involved in skimmer interaction leave the beam).

A satisfactory theory for (a) was published by Ashkenas and Sherman in 196* (1). A number of authors have also adequately studied (c), e.g. Zapata et al in 1960 (8). A summary appears in Hagena and Morton (7). Thus far (b) has eluded a satisfactory theoretical discussion It is the purpose of this work to shed some light on this dark area in the theory of nozzle beams.

DESCRIPTION OF THE PROGRAM

§1. METHOD USED

The Monte Carlo method used in this study of skimmer interaction is basically the method of G. A. Bird which has been described by him in a number of publications, for example (2). We include a short description here for sake of completeness and to enable us to explain the changes we have introduced to accommodate the peculiarities of this problem.

Bird's method is a Monte Carlo Algorithm for constructing a statistical sample of given size N of the the molecular paths occurring in some finite regions of an actual flow. The construction process is an imitation of what occurs in nature, at least insofar as probability distributions are concerned. The particle paths are functions of time, so that at any instant only the N coordinates in phase space (position, velocity-space) appear in computer memory. A time counter T is also kept in memory.

During each iterate of the calculations T is advanced by a preset quantity DTM (actually a number DT approximately equal to DTM) and the coordinates in the molecule list are updated.

§2. INITIAL CONDITIONS

The initial sample is from a distribution based on what we would have if the skimmer absorbed every molecule which hit it. Actually, the initial distribution is of secondary importance since experience has shown that the molecule list approaches the correct steady state distribution in relatively few iterations.

As soon as T reaches a preset value TEQ (time at which steady flow is assumed to have been achieved) data collection for subsequent output is begun and continues until T reaches TEND. Following output the program automatically stops or does the next case.

§3. ONE ITERATION

Each iteration has two components:

(a) Changes to molecule list due to collision.

(b) Changes to molecule list due to convection.

The computation of (a) requires that the flow region be divided into cells (Fig. 2) in such a way that the distribution function remains fairly constant over each cell. At any moment the molecules residing in a given cell constitute a sample from the distribution for that cell. If these molecules are allowed to collide among themselves then these collisions are representatives of the collisions actually taking place in the given cell. The theory backing this procedure is stronger than the above suggests. We refer to Bird's original papers on the subject.

The details of the collision process are as follows: A pair of molecules is chosen in such a way that the probability of being chosen is proportional to the relative velocity. If V_a and V_b are the velocities of the pair chosen, they are replaced according to the hard sphere collision law with V_a', V_b', where

$$V_a = 1/2[\bar{v}_a + \bar{v}_b - |\bar{v}_b - \bar{v}_a| \bar{n}]$$

$$V_b = 1/2[\bar{v}_a' + \bar{v}_b + |\bar{v}_a - \bar{v}_b| \bar{n}],$$

where $\bar{n}$ is chosen from the uniform distribution on the unit sphere.

The position coordinates remain unchanged and a time parameter DT is advanced by,

$$DDT = \frac{TCONS}{n^2 |\bar{v}_b - v_a|}.$$

The number TCONS is fixed for each cell and is explained in §6. Collisions are computed for a given cell until

$$DT + DDT > DTM.$$

The last collision is kept with probability

$$\frac{DTM - DT}{DDT}$$

and ignored otherwise. This last refinement is essential here because some cells have such low collision frequencies that no collisions at all should be computed during some intervals DTM.

The computation of (b) is basically simple. The position coordinates are translated on the basis of the time interval DTM and velocities. The computation involves some work because we use cylindrical coordinates in the program in which the translation equations are nonlinear. We have had to introduce two additional complications which do not appear necessary at first sight - referencing the molecule list §9 - and stock splitting §5. At the end of the collision convection cycle T is incremented appropriately and the whole process is repeated.

§4. GEOMETRY

Since the skimmer problem has cylindrical symmetry about the common axis of the free jet and the skimmer, two coordinates suffice to specify position - the axial and radial. They are denoted by (x,y) in the program (rather than (z,r) as is customary because essentially the same program with a few changed subroutines can be used in Cartesian coordinates). The velocity coordinates (u,v,w) are with respect to the Cartesian frame $\bar{i},\bar{j},\bar{k}$ where $\bar{i}$ is in the axial direction, $\bar{j}$ is in the radical direction and $\bar{k}$ is in the direction of increasing θ. The actual region of the flow under study is the solid of revolution generated by the shape in Fig. 1.

The flow region is subdivided into cells as required in the collision calculation. The 25 cells and the parameters necessary to specify the geometry are marked in Fig. 1. These parameters are kept in common area (GEOM) (see Appendix 1). Some of them must be specified initially (see Block Data subprogram) and the rest are computed in subroutine SETUP.

§5. STOCK SPLITTING

If the coordinates in computer memory simply constituted a sample of the molecular coordinates in an actual flow, we should find that their density in the x,y-plane increased linearly with x. This would be unsatisfactory because then most of the computation would be done for the region far from the axis while

the statistical scatter for the beam region containing only a small number of molecules would render these results almost useless.

The solution we have adopted to this difficulty involves subdividing the flow region into a number of subregions, using the streamlines of the flow to construct the boundary surfaces whenever possible. The subregions are then assigned weights in such a way that a simulator molecule in a subregion far from the axis "represents" several times as many "real" molecules as one in an axial region. For programming reasons we have found it convenient to make the weights inversely proportional to the representation multiples, so that the subregions farthest from the axis have the lowest weights.

Such an arrangement calls for a special strategy when a molecule happens to move between subregions of different weights. The strategy is this: When a molecule moves from a weight W_1 region to a weight W_2 region, it is replaced with

$$INTR(W_2/W_1)$$

identical molecules. The random variable INTR is defined as follows:

$$INTR\ (\alpha) = \begin{cases} [\alpha] & \text{with probability } 1-\alpha+[\alpha] \\ [\alpha] + 1 & \text{with probability } \alpha-[\alpha] \end{cases}$$

where $[\alpha]$ denotes the greatest integer less than or equal to α as is customary. We have called this process "stock splitting".

§6. TCONS CALCULATION

The original formula of G. A. Bird for the time advance per collision is $DDT = 2\Omega_k/(a_k n_k^2 v)$ where Ω_k, n_k, a_k are respectively the volume, total number of simulator molecules, and cross-section per simulator molecule for cell k, and v is the relative velocity of the collision pair. When the cells are weighted then a_k must be inversely proportional to the weight. That is if the weights are W_k, then

$$a_k\ W_k = const = \lambda.$$

The additional requirements on a_k in an actual calculation are

(1.) The cross-section density in a specified region of the flow is given (see §10).

(2) The total number of molecules must be in the vicinity of some N_0 smaller than the capacity of the molecule list.

Let us say N_0 is the number of molecules obtained in the flowfield if the skimmer caused no disturbance, i.e. absorbed everything that hit it. Let A_k be the total cross-section ((cross-section/molecule) x number of molecules) in cell k. A_k can be computed from the free jet structure and the cross section density A_c at the skimmer orifice (subroutine VPOL). Then

$$N_k = \frac{A_k}{a_k} = \frac{A_k W_k}{\lambda}$$

and

$$N_0 = \frac{1}{\lambda} \sum_{k} A_k w_k ,$$

so that

$$\lambda = \frac{1}{N_0} \sum A_k w_k$$

and

$$TCONS(\kappa) = 2\Omega_k/a_k = 2\Omega_k w_k N_0/\Sigma A \ w \quad , \ DDT = \frac{TCONS(\kappa)}{n_\kappa^2 v} .$$

§7. THE INCOMING DISTRIBUTION

First the total influx rate CU to the flow region from the jet is computed from the jet cross-section density A_e at the entry point, its velocity v (taken = 1 in our units) and the weights of the cells receiving the molecules. The total entering cell κ (assuming all k borders on the entry plane) is

$$\frac{A_e v}{a_\kappa} \times \text{area of entry surface element.}$$

CU is the sum of all such quantities.

The number entering the flow region during time DT is CU x DT. Each entering molecule has its position coordinates chosen at random in accordance with the Ashkenas-Sherman formula [1] for variation of jet density with distance from the axis

$$\frac{\rho(R,\theta)}{\rho(R,o)} = \cos^2 \left(\frac{T \ \theta}{2\phi}\right)$$

and the weight of the cell about to receive the molecule. The details of this calculation appear in subroutine INPUT (Appendix 1).

§8. BOUNDARY CONDITIONS

The boundaries in question are the skimmer walls and the artificial boundaries introduced to enclose the flow field. A molecule colliding with a skimmer has a choice of being reflected or reemitted from a cosine distribution at a specified temperature CR. The probability of reemission is a specified accommodation coefficient ACCOM.

The artificial boundaries are:

(1) The entry plane,
(2) The upstream outer boundary,
(3) The downstream boundary.

Molecules which collide with (1) and (3) are deleted. If a molecule collides with (2) its velocity vector is reflected and augmented by the vector 2pn where n is the outward normal to the wall the p is a specified constant. If the resulting velocity vector points out of the region the molecule is deleted; otherwise it

is allowed to reenter. When p is roughly equal to the normal component of the average velocity of the molecules crossing the wall, its net effect is to simulate no discontinuity in the flow due to the boundary.

§9. REFERENCING THE MOLECULE LIST

We discuss here the actual arrangement of the molecule coordinates C_d d = 1,2,...in computer memory consistent with a fast method for referring to the coordinates corresponding to a given cell.

The list is arranged in such a way that

$$C_1 \quad \cdots \quad C_{N1}$$

are in cell 1,

$$C_{N1+1} \quad \cdots \quad C_{N2}$$

are in cell 2, etc. The numbers NI are stored in an auxiliary list called MAP. The basic operations which must be performed on this list are

    (1) Removal of a molecule, and
    (2) Addition of a molecule.

(1) To remove C_α where $N_{i-1} < \alpha \leq N_i$, store C_{N_i} in place of C_α, $C_{N_{i+1}}$ in place of C_{N_i}, etc. until the end of the list is reached.

(2) To insert coordinates C into cell i we make room for it by storing the first molecule of each cell in the location following the last molecule, starting from the end and going up to cell i + 1. This leaves location N_i + 1 for C.

Of course, in each case MAP is updated to reflect the new configuration.

It is clear that a simple transfer of molecule from one cell to another does not require the operations to go to the end of the coordinate list. A time saving procedure to accomplish this task is incorporated into the program.

§10. SIMULATING AN ACTUAL EXPERIMENT

The program takes the limiting mean centreline jet velocity to be 1. All other velocities are given in terms of it. The simulator geometry must be similar (in the geometric sense) to the real geometry, but the units are arbitrary. Once the velocity and the distance unit are fixed, the time unit is automatically determined. Since we are mostly concerned with the steady state we do not even need to compare computed times with real times except in the initial approach to the steady state.

The remaining parameter that must be assigned is the cross section density A at some reference point in the flow which we have chosen to be the skimmer orifice assuming perfect skimming. A has units of reciprocal length, in fact it is just a multiple of 1/mean free path. Thus Ad, where d is the skimmer orifice diameter is dimensionless, and we give it the same value in the simulator as it has in the experiment.

Though this prescription is simple, it has its problems, not the least

being that the hard sphere cross section of real molecules depends on their relative velocity. The best we can do is to use, some sort of average relative velocity. The realtive velocity of the beam and the molecules coming off the skimmer wall is a good candidate because this is when most of the collisions take place.

§11. OUTPUT FORMAT

The output from each run contains the following information (see Appendix 2).

(1) A printout of DT and MAP for each interval DTM for checking purposes.

(2) A flow density profile at the skimmer orifice as well as a total particle flow rate.

(3) A flow density profile at the far downstream end of the flowfield as well as the total particle flow rate through the geometrical beam area. This last figure is useful in estimating the centreline beam intensity

(4) Histograms for the integrated distribution function profiles ∫fdvdw, ∫fdwdu, ∫fdudv for each cell.

The distribution function is broken down into the distribution function of the molecules which have not undergone collision in the flowfield and the remainder. The mean velocity, mean square velocity, and mean number of molecules are also given along with a theoretical comparison number for no skimmer interaction. The histograms are obtained by time averaging regular observations of the flowfield for the duration of the computation after equilibrium is reached

RESULTS

A simulator program such as this one should be viewed rather like an experimental facility suitable for investigating a range of problems connected with the skimmer. We describe here the results of a study of skimmer interaction.

A preliminary study was done to check a model proposed by French (5) suggesting a significant interaction downstream of the skimmer orifice. This was done in a program which preceeded the present one by making the downstream skimmer wall absorbing. No downstream effect was observed. A check of the distribution functions in the present study supports this conclusion.

In fact this study shows that the skimmer interaction is caused by a screen of molecules originating from the wall and from collisions extending approximately one mean free path upstream of the skimmer orifice The supporting material for this conclusion is contained in the following graphs and tables

Table 1 shows the cases computed in this study.

Fig. 1 defines the geometric variables in the program and Figs 2 - 4 give the specific geometries used in the calculations.

Table 2 shows condition near the skimmer wall for an assortment of cases. It is most significant that for all but the sharpest skimmer angles 50°, 45° the flow is subsonic. We suspect there is always a subsonic region close to

the skimmer wall which is very thin for small angles but grows rapidly as
the angle increases. Also the surface interaction with the skimmer plays
a significant role, since case 23 (ACCOM = 0) gives a high Mach number even
for a blunt skimmer.

Fig. 5 shows the mass flow rate through the skimmer orifice vs A in
terms of the theoretical no interaction value. The mass flow is less than
one in all cases computed but shows a tendency to go to 1 in the cold
skimmer and no lip cases.

Fig. 6 shows beam intensity vs A. The importance of the skimmer lip
sharpness is here clearly indicated. The skimmer angle seems to have less
effect than the skimmer temperature. Though the general shape of the exper-
imental beam intensity vs Knudsen number curves is evident, our results do
not show decreases in intensity as strong as those in [3] for instance.

Figs. 7, 8 show mass flow and beam intensity vs various parameters. It
is most interesting that an increase in PRES which leads to a decrease in the
number of molecules entering the flowfield through the outer upstream boundary
leads to a decreased mass flow but no significant change in beam intensity.
Furthermore, a low accommodation coefficient gives high values in both graphs.

Figs. 9, 10 were obtained by smoothing the histograms produced by the
program. It is clear that a Knudsen number of 1/2 (A = 4) leads to more or
less complete destruction of the beam by skimmer interaction.

Figs. 11, 12 show clearly a sharp increase of molecules having a broad
distribution function and infiltrating the beam region between Knudsen numbers
1/2 and 1.

Finally a note on the standard deviation of the results. The beam intensity
and mass flow curves have a relative standard deviation of between 3% and 5%.
It follows from the theory of Monte Carlo simulators that the probability distri-
bution of each value obtained is normal. Accuracy is expensive in these calcul-
ations. A fourfold increase in computing time would cut the standard deviation
in half.

TABLE 1

CASES COMPUTED

	A	SIA	CR	PRES	ACCOM	THLIP	GEOM	COMMENTS
1	0.5	0.1	1.	1.	0.5	0.1	3	Sharp skimmer
2	1.	0.1	1.	1.	0.5	0.1	3	
3	2.	0.1	1.	1.	0.5	0.1	3	
4	4.	0.1	1.	1.	0.5	0.1	3	
5	2.	0.1	1.	2.	0.5	0.1	1	Effect of PRES
6	2.	0.1	1.	1.5	0.5	0.1	1	
7	2.	0.1	1.	0.75	0.5	0.1	1	
8	2.	0.1	1.	0.5	0.5	0.1	1	
9	2.	0.0546	1.	1.	0.5	0.1	1	High mach number
10	1.	0.1	1.	2.	0.5	0.1	1	Absorbing walls
11	0.5	0.1	0.5	1.	0.5	0.1	1	Cold skimmer series
12	1.	0.1	0.5	1.	0.5	0.1	1	
13	2.	0.1	0.5	1.	0.5	0.1	1	
14	4.	0.1	0.5	1.	0.5	0.1	1.	
15	0.5	0.1	1.	1.	0.5	0.1	1	Standard series
16	1.0	0.1	1.	1.	0.5	0.1	1	blunt skimmer
17	2.0	0.1	1.	1.	0.5	0.1	1	
18	4.0	0.1	1.	1.	0.5	0.1	1	
19	0.5	0.1	1.	1.	0.5	0.0	1	Perfect edge
20	1.0	0.1	1.	1.	0.5	0.0	1	
21	2.0	0.1	1.	1.	0.5	0.0	1	
22	4.0	0.1	1.	1.	0.5	0.0	1	
23	2.	0.1	1.	1.	0.	0.1	1	Effects of surface
24	2.	0.1	1.	1.	1.	0.1	1	interaction
25	0.5	0.1	1.	1.	0.5	0.1	2	Intermediate angle
26	1.	0.1	1.	1.	0.5	0.1	2	
27	2.	0.1	1.	1.	0.5	0.1	2	
28	4.	0.1	1.	1.	0.5	0.1	2	

Notes   A vs Knudsen number Kn

A	0.5	1	2	4
Kn	4	2	1	0.5

SIA, CR, PRES, ACCOM, THLIP are explained in Appendix .

GEOM 1, 2, 3 are shown in Figs. 1, 2, 3 respectively.

TABLE 2

CONDITIONS IN FRONT OF SKIMMER

	CELL 14			CELL 15		
Case	Mean Speed	Mach #		Mean Speed	Mach #	Inclination
15	0.55	0.64		0.51	0.55	0.83
18	0.52	0.79		0.34	0.44	0.69
5	0.62	0.88		0.48	0.57	0.52
8	0.33	0.44		0.28	0.34	0.73
1	0.80	1.4		0.72	1.2	0.28
4	0.86	2.1		0.71	1.	0.32
23	0.87	2.2		0.75	2.3	0.62
25	0.68	1.0		0.60	0.82	0.43
28	0.71	1.3		0.53	0.73	0.47

REFERENCES

1. H. Ashkenas and F. S. Sherman, The Structure and Utilization of Supersonic
    Free Jets in Low Density Wind Tunnels, p. 84, Supplement 3, Rarified Gas
    Dynamics, New York, 1969.

2. G. A. Bird, Direct Simulation Monte Carlo Method, p. 85, Vol. 2, Supplement
    5, Rarifield Gas Dynamics, New York, 1969.

3. U. Bossel, F. C. Hurlbut, and F. S. Sherman, Extraction of Molecular Beams
    from Nearly Inviscid Free Jets, p. 945, Vol. 2, Supplement 5, Rarified
    Gas Dynamics, New York, 1969.

4. R. Campargue, Dimensionless Number Linked to Background and Skimmer Jet
    Interaction in Nozzle Beam Generation, p. 1003, Vol. 2, Supplement 5,
    Rarified Gas Dynamics, New York, 1969.

5. J. B. French, AGARD ograph 112 (N.A.T.O.) April 1966.

6. T. R. Govers, R. L. LeRoy, and J. M. Deckers, The Concurrent Effects of
    Skimmer Interaction and Background Scattering on the Intensity of a
    Supersonic Molecular Beam, p. 985, Vol. 2, Supplement 5, Rarified Gas
    Dynamics, New York, 1969.

7. Hagena, and Morton, Analysis of Intensity and Speed Distribution of a
    Molecular Beam from a Nozzle Source, p. 1370, Supplement 4, Rarified Gas
    Dynamics, New York, 1969.

8. I. N. Zapata, H. M. Parker, and J. H. Bodine, Performance of a Supersonic
    Molecular Beam, p. 67, Supplement 1, Rarified Gas Dynamics, New York, 1969.

FIG. 1 DEFINITION OF GEOMETRICAL PARAMETERS
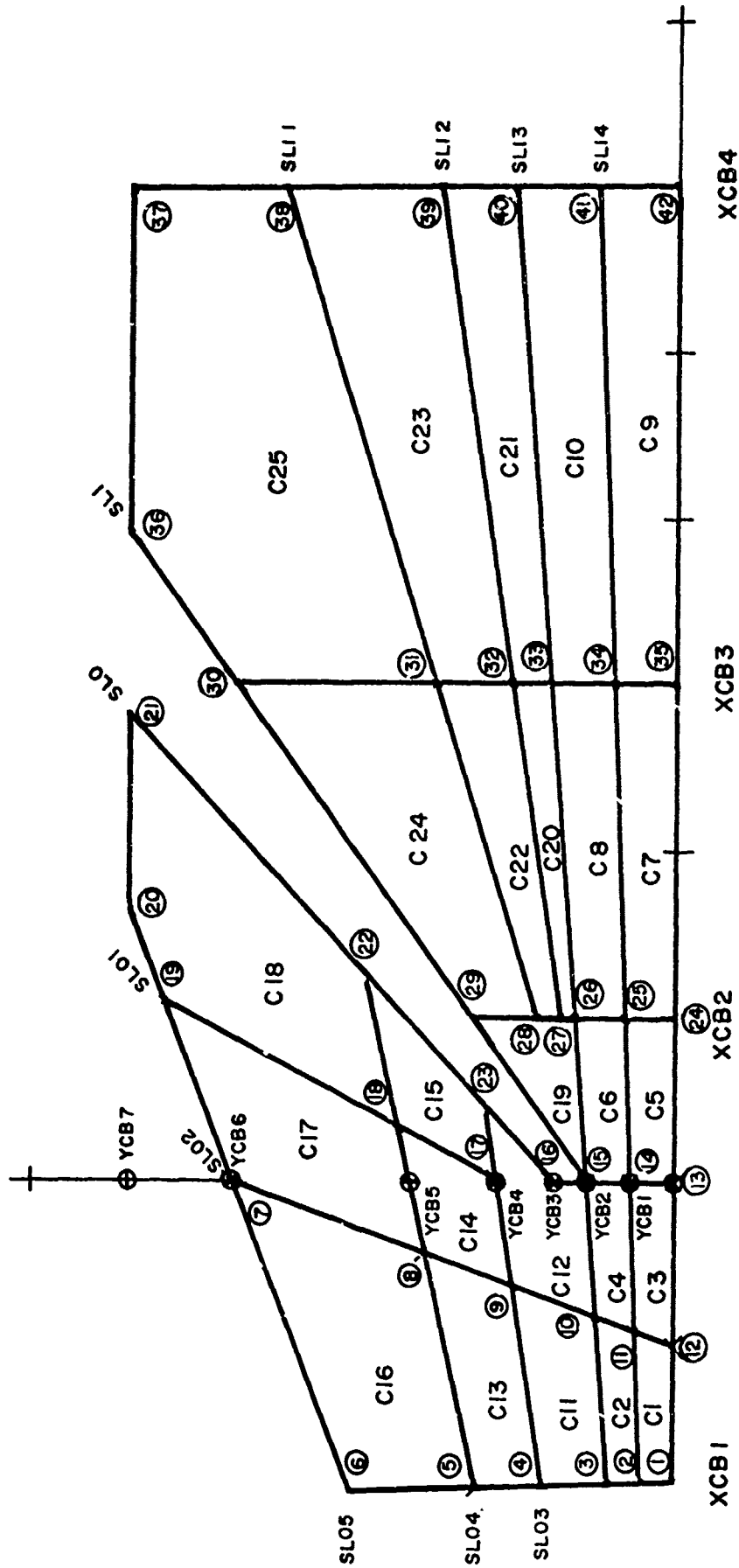
KEY: SLI, ..., SL05 SLOPES
         XCB1, ..., YCB4   X & Y INTERCEPTS RESP.
         ①, ..., ㊷ VERTEX NUMBERS
         C1, ..., C25 CELL NUMBERS

FIG. 2 FLOWFIELD FOR GEOM = 1, SKIMMER ANGLES 96°, 70°
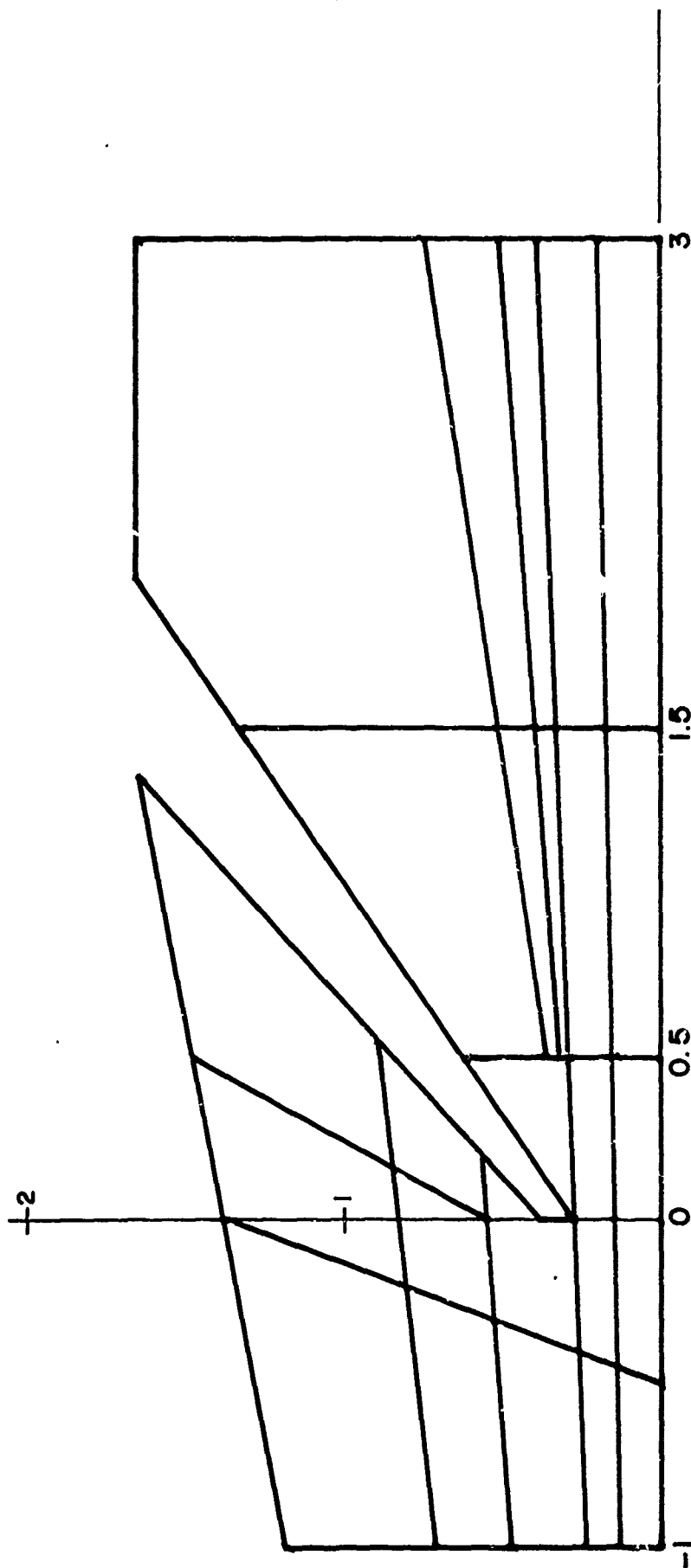
FIG. 3 FLOWFIELD FOR GEOM = 2, SKIMMER ANGLES 68°, 53°
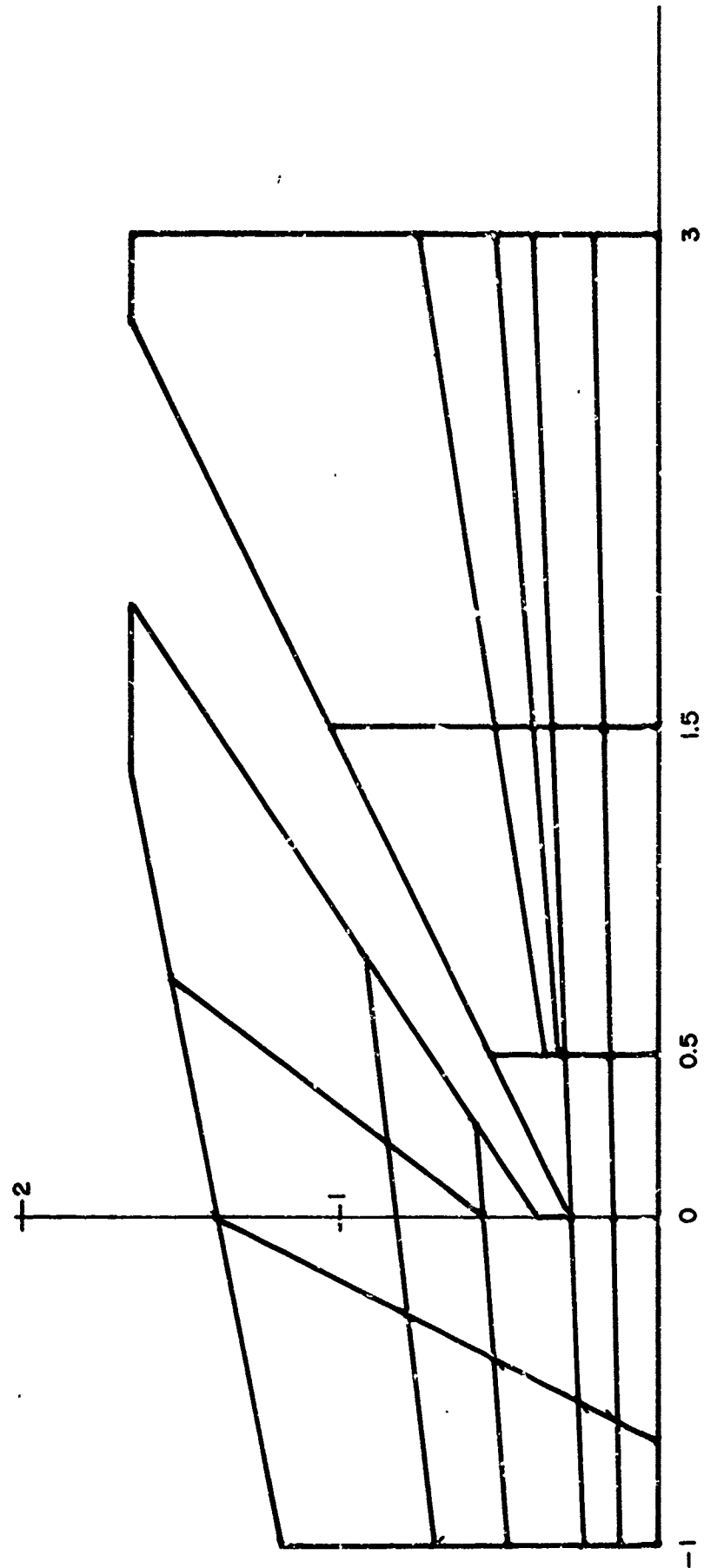
FIG. 4 FLOWFIELD FOR GEOM = 3, SKIMMER ANGLES 50°, 40°
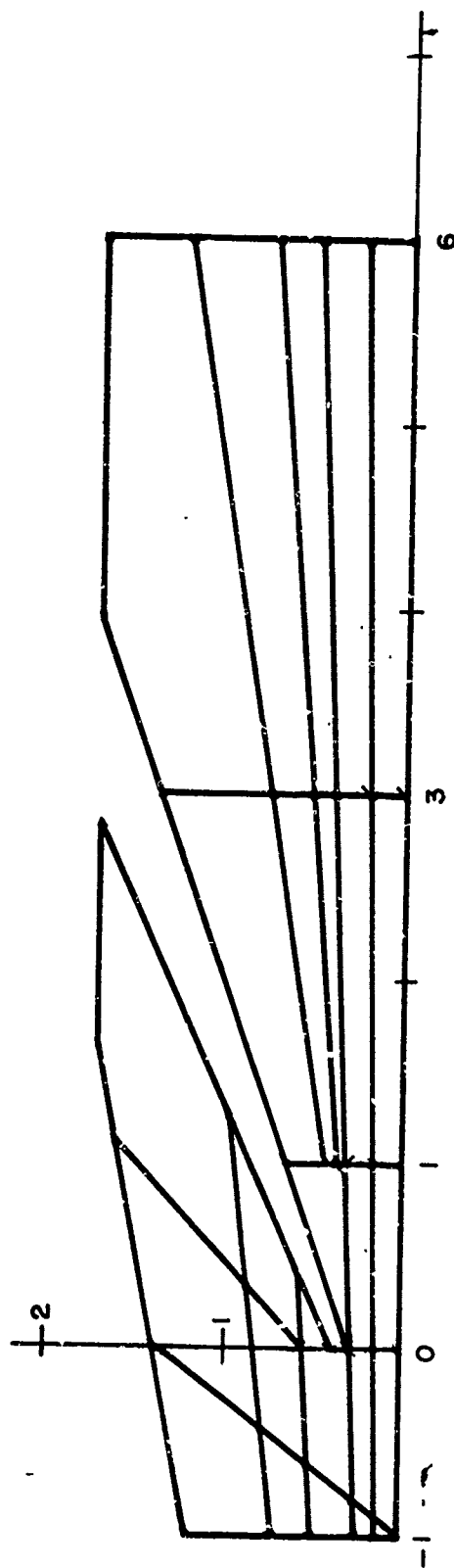
FIG. 5 MASS FLOW THROUGH SKIMMER UNDER VARIOUS CONDITIONS

Sharp Skimmer (3)

Cold Skimmer

Intermediate Skimmer (2)

Blunt Skimmer (1)

No Lip
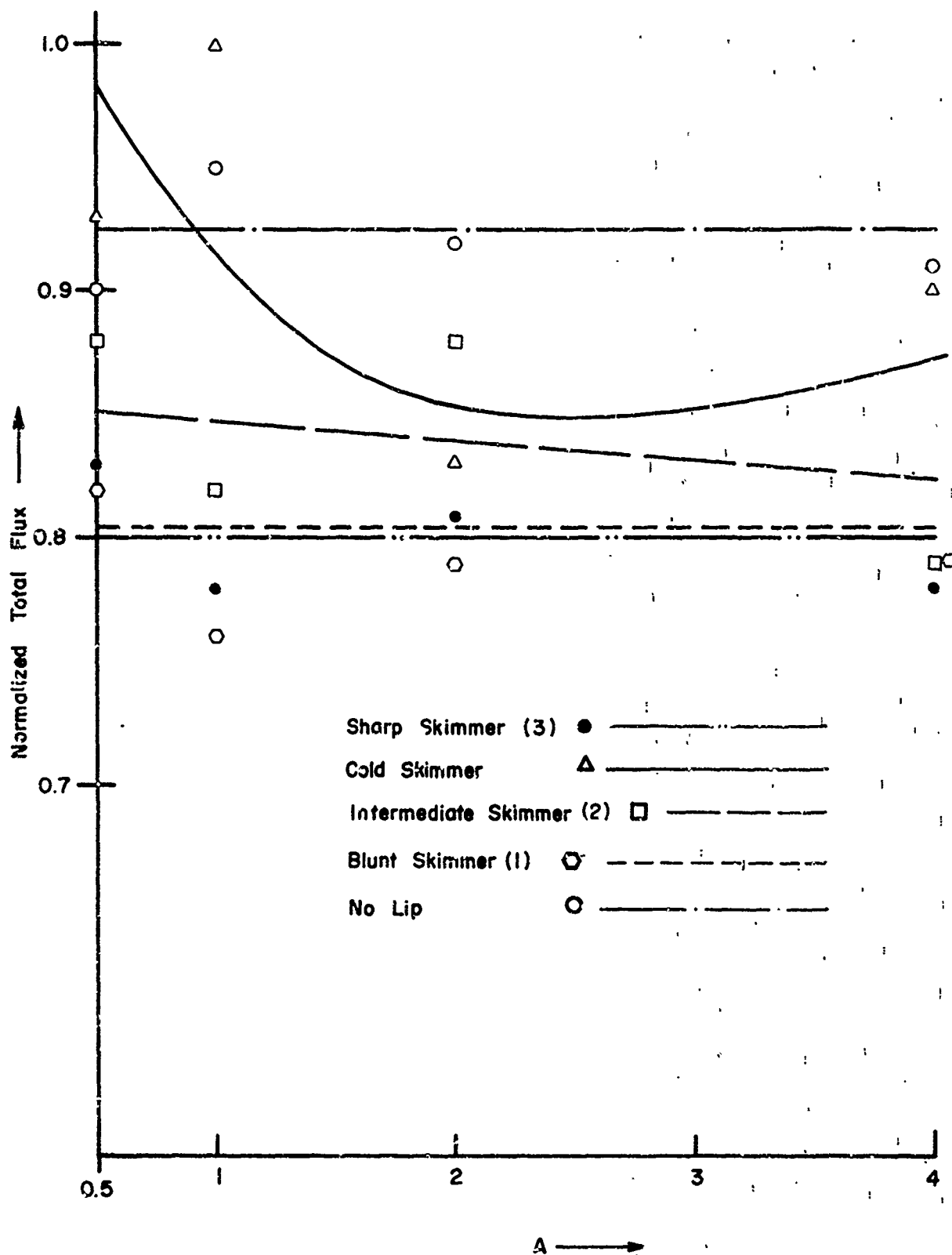
FIG. 6 BEAM INTENSITY UNDER VARIOUS CONDITIONS
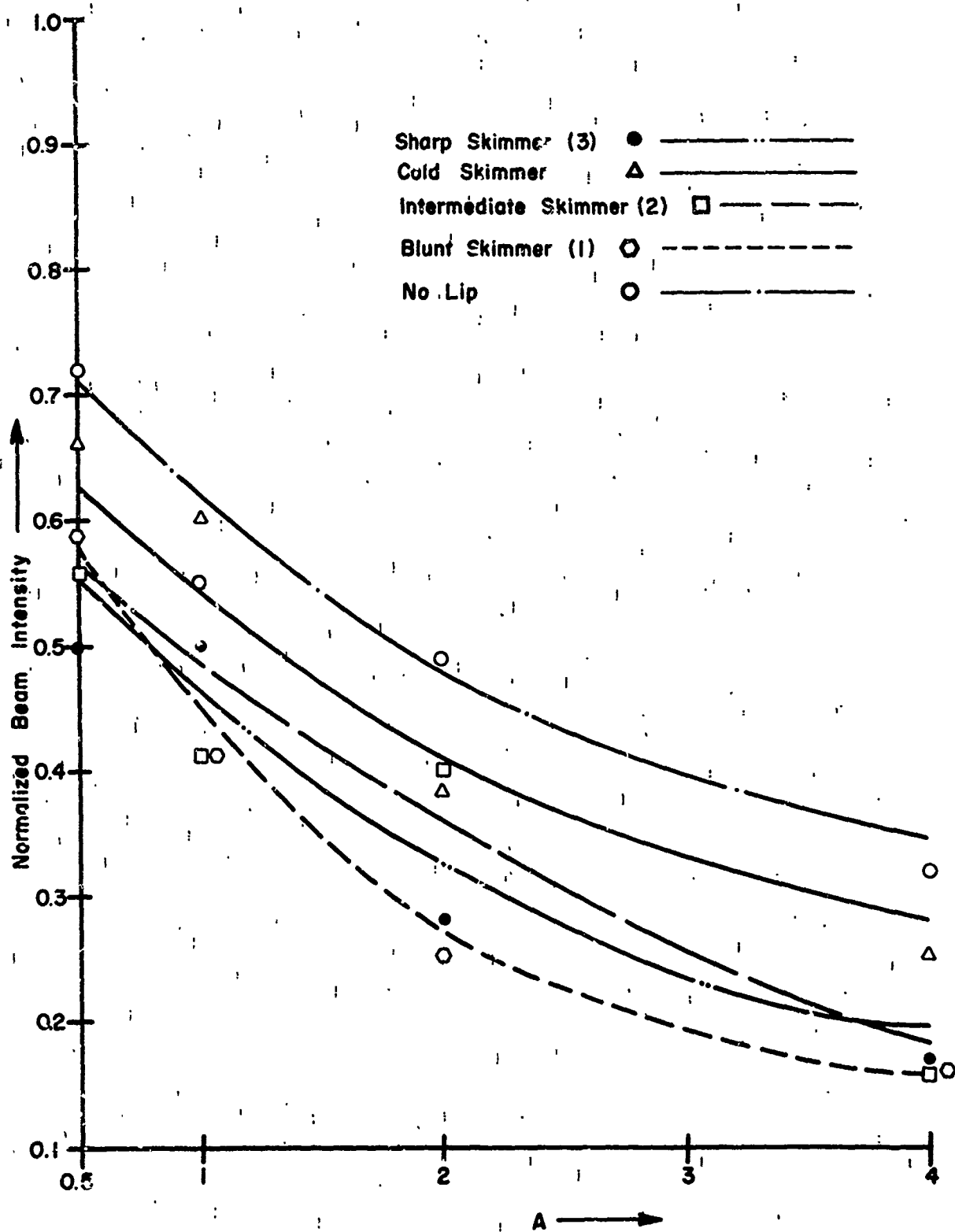
FIG. 7 MASS FLOW THROUGH SKIMMER VS OTHER PARAMETERS
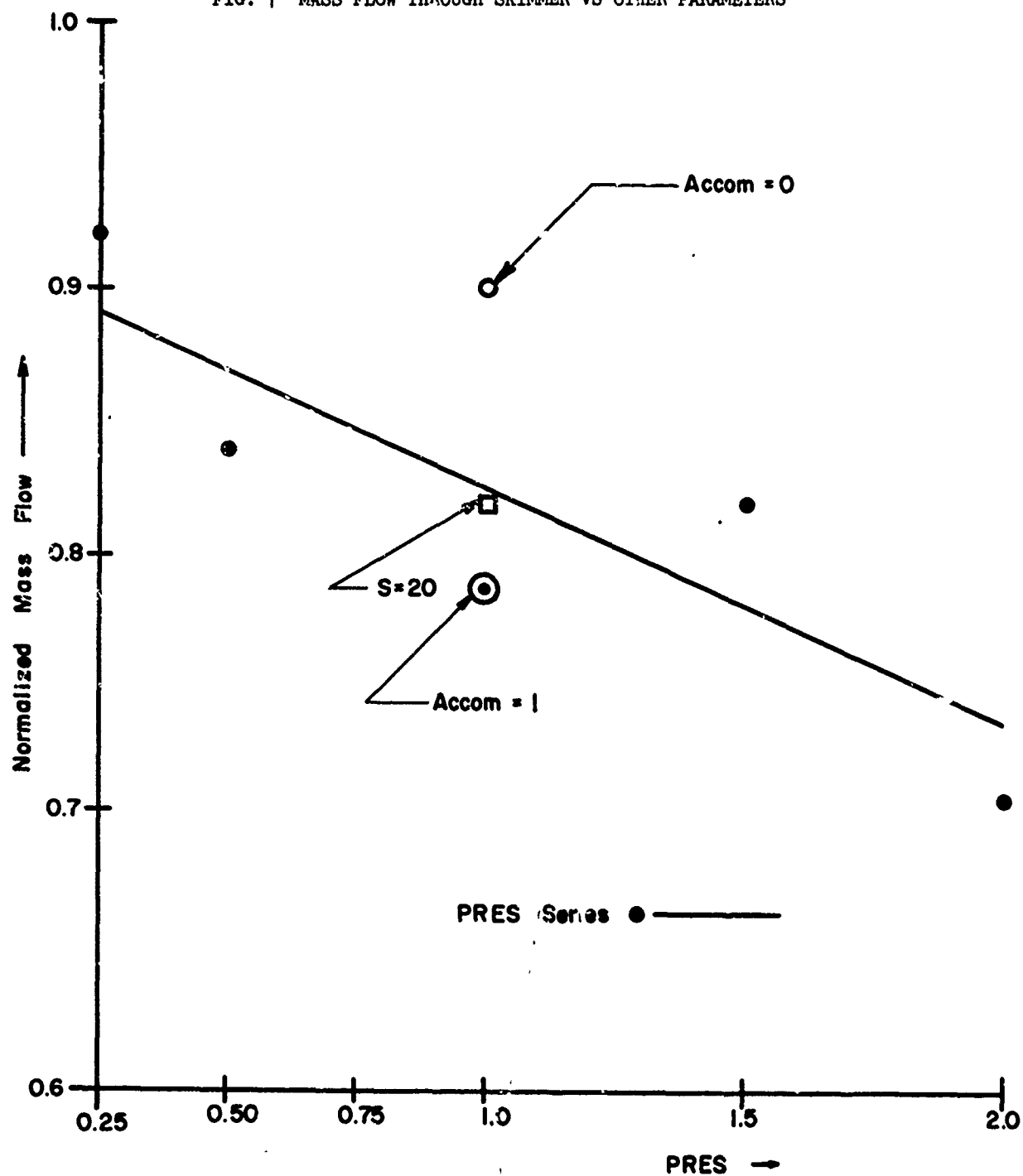
FIG. 8   BEAM INTENSITY VS OTHER PARAMETERS
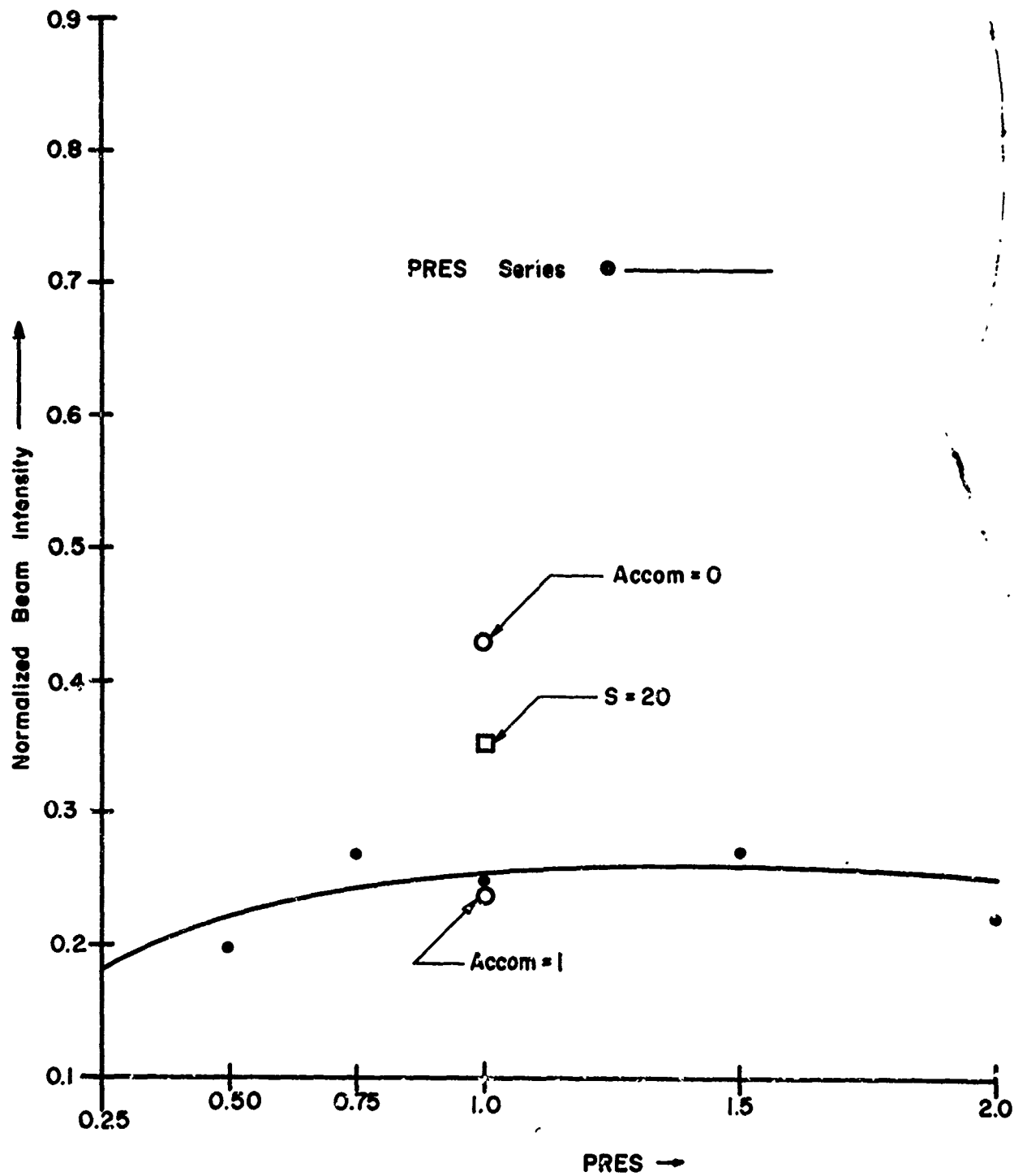
FIG. 9 FLUX DENSITY AT SKIMMER ORIFICE (GEOM=2)
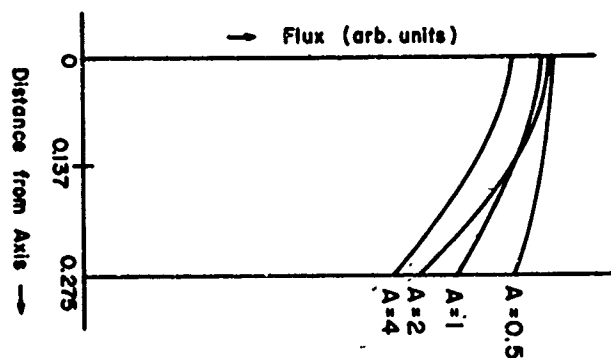
FIG. 10 BEAM PROFILES (GEOM=2)
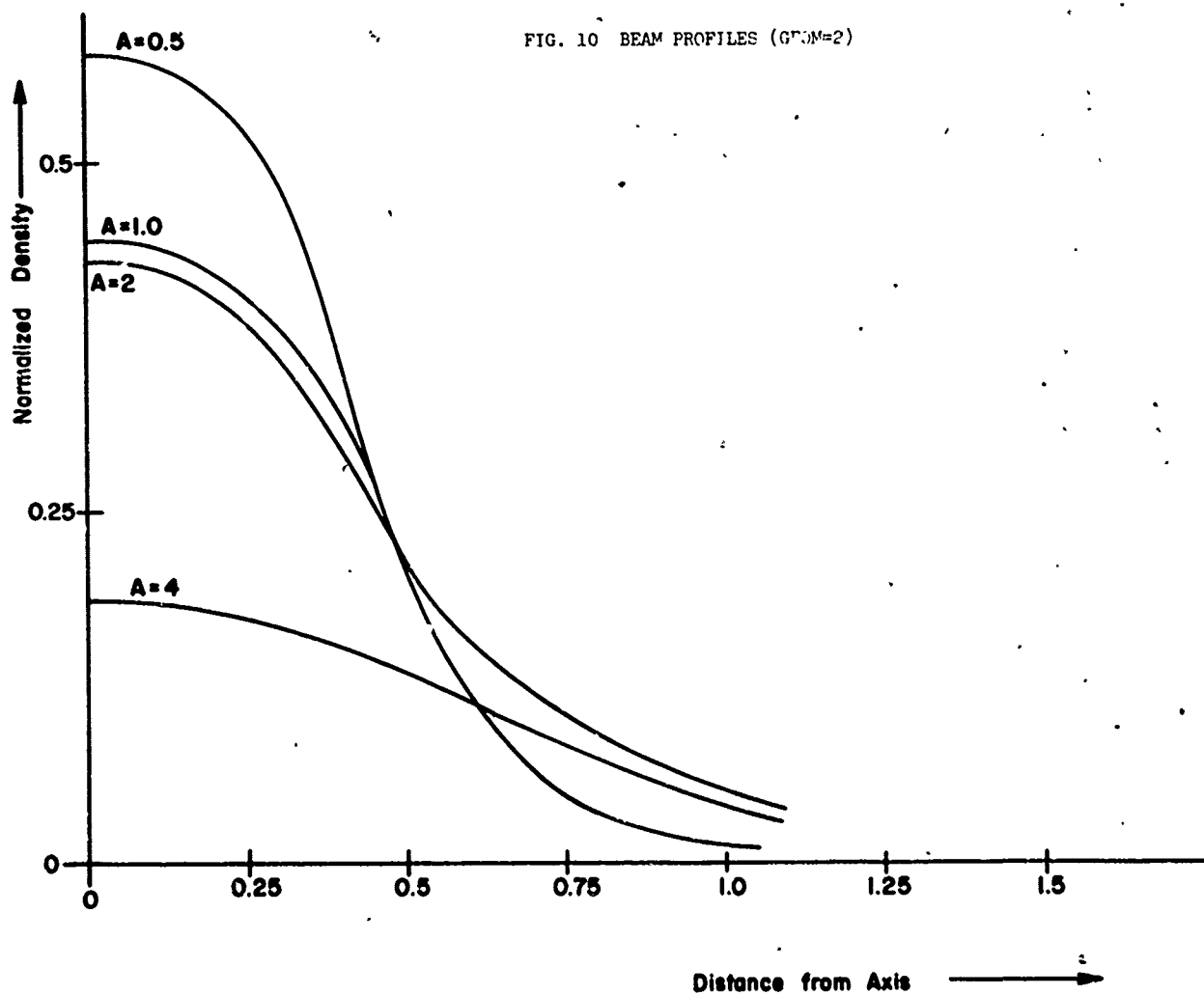
FIG. 11 DISTRIBUTION FUNCTION IN CELL 4 (GEOM=2)
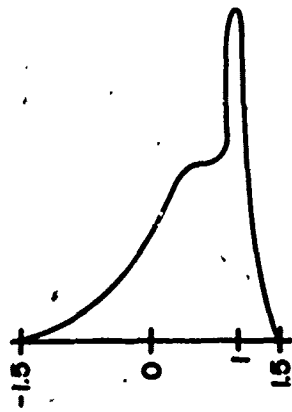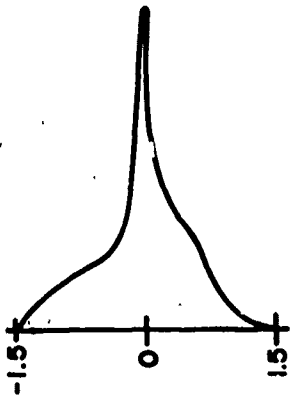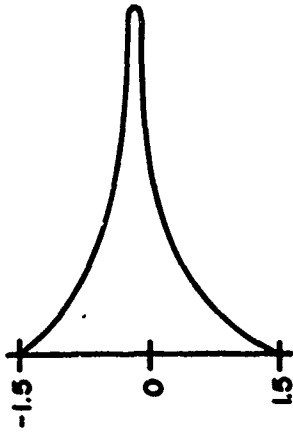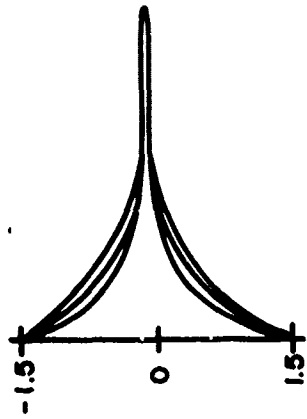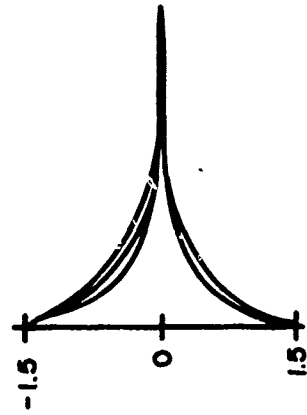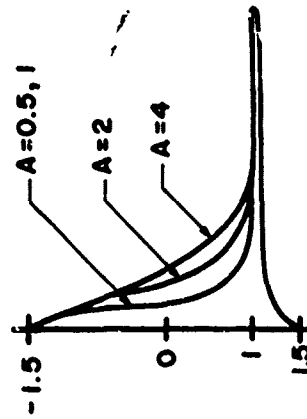
FIG. 12 CONDITIONS JUST UPSTREAM OF ORIFICE (GEOM=2)
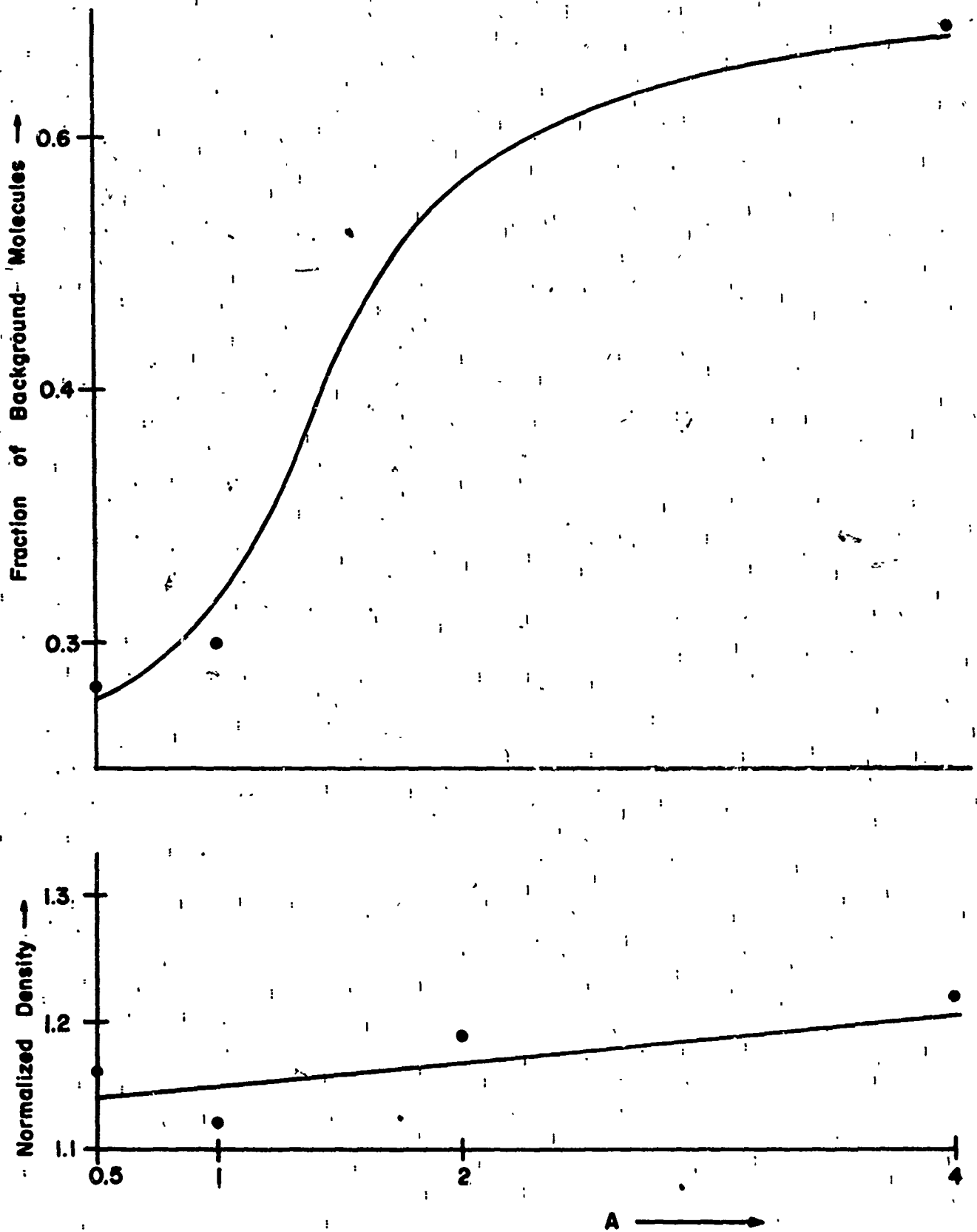
COMMENTS

A. The Variables

Common Area	Purpose
/RCRD/	Storage area for beam and orifice profile histograms. BPRFL and HPRFL resp. DBPR and DHPR are the widths of the histogram cells in each case.
/GEOM/	Geometrical definition of the flow field - see Fig.
/NPRFL/	For internal use in the program to save computer time in the introduction of molecules through the input plane.
/CONS/	CU is the total number of molecules entering the flow field through the input plane per unit time, SIA is the speed ratio, in the jet, CR is a speed defining the temperature of the skimmer walls, ACCOM is the accommodation coefficient, PRES is related to the pressure on the upstream outer boundary, CLWT is the list containing the cell weights, TCONS is the list of time advance per collision constants for the cells. The remaining variables are for internal use of the program to save recomputing frequently used numbers. They are initially computed in SETUP.
/BEGIN/	BE and BG are initial values for the sequencing variables of the random number generators. A is the cross section density at the orifice, and NST is starting number of simulator molecules in the flow field.
/TIME/	T - Accumulated time variable for the program.
	DT - Weighted time per iteration ($DT \sim DTM$).
	DTM - Preset time allowance per iteration.
	TEQ - Value of T for which equilibrium is assumed to have been achieved.
	TEND - Value of T at which computation ceases and output is begun.
	The remaining variables were not actually used in this program.
/STORE/	NCL is the number of cells, MCP = 5000, and MAP is for purposes of referencing the list of molecules. The remaining variables in this common area constitute the molecular list.
/HIST/	Storage area for the histograms FU etc; CFU etc; the mean velocity vectors for each cell - UB etc. and the mean square velocity vector for each cell UUB - CORG counts the number of molecules in each cell which have undergone more than one collision.
/SW/	Contains control parameters for the data collection operation.

/RAND/   Contains the sequencing parameters for the random number generators.

/VARS/   Temporary storage area for one molecule to save address computation time while the molecule is being processed.

B. Subroutines

MAIN   This program is used to set up the cases. Each call to PROG constitutes the computation of one case.

BLOCK DATA   Used to assign values to variables which are common to the computation of many cases.

PROG   Control program for the computation of one case.

SETUP   Computes variables not assigned in MAIN or BLOCK DATA and performs all the initialization necessary to begin computation of a case.

VPOL   Computes cell volumes and cell masses for the undisturbed free jet.

INPR   Sets up an array used to obtain the input profile by interpolation.

LOC   Function which computes the cell containing the coordinates (x,y). LOC works by computing a word IX which has 0 or 1 in the k-th bit depending on which side of the k-th boundary (x,y) is found. Then IX is compared with a standard ordered list (MASK, 1DFR).

COLI   Collision increment for cell L.

SQM   Estimates the maximum relative velocity in a given cell.

TRBKP   "Translate and Book-keep". This subroutine essentially controls computation of the convection increment.

INVRS   Computes the cell number L of the M-Th molecule in the molecule list.

UP, DOWN   Used in inserting, deleting or moving molecules in the molecule list.

TRS, TR   Transfer of coordinates.

FRP   Checks a given molecule for collision with boundaries and it moves it a distance appropriate to the time DT.

TRANS   Changes coordinates of a given molecule appropriate to the time interval DT.

LCHK   Checks whether collision occurs with a given boundary.

EMIT   Emits a fully accommodated molecule in the direction C,S.

REFL   Reflects a molecule in the direction C,S.

INPUT   Introduces CU and DT molecules through the input plane on the basis of the input profile defined by FNPR, DFNPR.

CBSPL   Cubic Spline interpolation subroutine.

TABU   Controls the accumulation of histograms and moments of the distribution function.

FIT   Inserts molecule at cell L.

SIFT   Auxiliary subroutine to accumulate histograms.

INTR   See §5.

RANDG   Standard normal random number generator.

RANDL   Uniform on $[0,1]$ random number generator.

RANDR   Uniform on $[-1,1]$.

UMAX   Finds the maximum element in an array.

ISGN $ISGN(X) = 1$ if $x \geq 0)$
                         0 otherwise

DUMP.   Controls the output function of the program.

BAR   Sets up one histogram bar for printing.

HIST1   Sets up one histogram for printing.

HIST3   Sets up and prints the 3 x 3 histogram describing the distribution function in cell L.

APPENDIX I

PROGRAM LISTING

```
//FORT.SYSIN DD *
      COMMON /XCRD/ DRPR,DHPR,RPRFL(50),HPRFL(10)
      COMMON /GEOM/ PFI,DISNS,DIAMS,THLIP,SLO,SLI,YCR1,XCB2,YCR3,YCR4,
     X      SLI1,SLI2,XCR0,XCR1,XCR5,YCR1,YCR2,YCR3,YCR4,YCR5,YCR4,YCR7,
     X SLO1,SLO2,SLO3,SLO4,SLO5, SLI3,SLI4, RADIN,DISMI, XNI,YNI,
     X XMO,YMO, XNS,YNS, VOL(25),VWD(25)
      COMMON /HPRFL/ ZNPR(10),FNPR(10),DFN*X(10)
      COMMON /CONS/ CU,SIA,SIR,CP,CT,ACCOM,PRES,CLWT(26),TCONS(25)
      COMMON /REGIN/ RR,RG,A,NST
      COMMON /TIME/ T,DT,DTM,TFO,DTSN,TFND,DTRP,TRFP
      COMMON /STORE/ NCL,NCP,MAP(30),X(5000),Y(5000),U(5000),V(5000),
     X W(5000),ID(5000)
      COMMON /HIST/ FU(10,25),FV(10,25),FW(10,25),CFU(10,25),CFV(10,25),
     1CFW(10,25),UR(25),VR(25),WR(25),IRR(25),VVR(25) WWR(25),COR*(25)
      COMMON /SW/ KSAM,RRFP
      COMMON /RAND/R,G
      COMMON /VARS/ XO,YO,XM,YM,UM,VM,WM,IUM
      INTEGER RR,RG
      RR=1795
      RI=1795
      NST=1000
      DTM=1./3.
      DTSN=1.
      TRFP=0
      DTRP=1.
      CM=1.
      ACCOM=0.5
      NCL=25
      NCP=3000
      TFO=4.
      TFND=12.
      SIA=.1
      YCR7=1.6427619
      SIA=0.1
      ACCOM=0.5
      A=2.
      PRES=0.5
      PRES=1.
      ACCOM=1.
      ACCOM=0.
      CALL PROG(1,1)
      PRES=1.
      ACCOM=0.5
      CR=1.0
      SLO=0.67
      SLI=0.5
      A=0.5
      CALL PROG(1,1)
      A=1.
      CALL PROG(1,1)
      A=2.
      CALL PROG(1,1)
      A=4.
      CALL PROG(1,1)
      STOP
      END
      BLOCK DATA
      COMMON /CONS/ CU,SIA,SIR,CR,CT,ACCOM,PRES,CLWT(26),TCONS(25)

      COMMON /GEOM/ PFI,DISNS,DIAMS,THLIP,SLO,SLI,XCR1,XCR2,YCR3,YCR4,
     X      SLI1,SLI2,XCR0,XCR1,XCR5,YCR1,YCR2,YCR3,YCR4,YCR5,YCR4,YCR7,
     X SLO1,SLO2,SLO3,SLO4,SLO5, SLI3,SLI4, RADIN,DISMI, XNI,YNI,
     X XMO,YMO, XNS,YNS, VOL(25),VWD(25)
      DATA PFI/1.15/,DISNS/7.0/,DIAMS/0.55/,THLIP/0.1/,SLI/0.5/,
     X SLI/0.7/,XCR1/-1./,XCR2/0.5/,XCR3/1.5/,XCR4/3.0/,CU/25./,
     X SIA/0.0546/,CR/1./,ACCOM/0.5/,PRES/.5/,
     X CLWT/10*1.,2*0.25,10.0625,100.01562,5*1...0./
      END
      SUBROUTINE PROG(NRPS,NARS)
C     SKIPPER 1970
      COMMON /VARS/ XO,YO,XM,YM,UM,VM,WM,IUM
      COMMON /SW/ KSAM,RRFP
      COMMON /TIME/ T,DT,DTM,TFO,DTSN,TFND,DTRP,TRFP
      COMMON /STORE/ NCL,NCP,MAP(30),X(5000),Y(5000),U(5000),V(5000),
     X W(5000),ID(5000)
      CALL SETUP(NRPS)
      IF (T-TRFP) 3,2,2
    3 RRFP=2
      TRFP=TRFP+DTRP
      GO TO 4
    2 RRFP=1
    4 IF(T-TFO) 6,5,5
    5 KSAM=2
      GO TO 7
    6 KSAM=1
    7 DT=0.
      DO 8 L=1,NCL
      CALL COL(L)
      DT=DT/FLOAT(MAP(NCL))
      WRITE(6,99) DT,(NZPR(I),I=1,25)
   99 FORMAT(' DT=',E7.5,'  MAP=',25I4)
      CALL THRAP(NARS)
      CALL INPUT
      GO TO (15,14),KSAM
   14 CALL TAMI
   15 CONTINUE
      T=T+DT
      IF (T-TFND) 1,9,9
    9 CALL DUMP
      RETURN
      END
      SUBROUTINE SETUP(NRPS)
C     SKIPPER 1970
      COMMON /HIST/ FU(10,25),FV(10,25),FW(10,25),CFU(10,25),CFV(10,25),
     1CFW(10,25),UR(25),VR(25),WR(25),IRR(25),VVR(25),WWR(25),COR*(25)
      COMMON /STORE/ NCL,NCP,MAP(30),X(5000),Y(5000),U(5000),V(5000),
     X W(5000),ID(5000)
      COMMON /HPRFL/ ZNPR(10),FNPR(10),DFN*X(10)
      COMMON /REGIN/ RR,RG,A,NST
      COMMON /RAND/ R,G
      COMMON /GEOM/ PFI,DISNS,DIAMS,THLIP,SLO,SLI,XCR1,XCR2,XCR3,YCR4,
     X      SLI1,SLI2,XCR0,XCR1,XCR5,YCR1,YCR2,YCR3,YCR4,YCR5,YCR4,YCR7,
     X SLO1,SLO2,SLO3,SLO4,SLO5, SLI3,SLI4, RADIN,DISMI, XNI,YNI,
     X XMO,YMO, XNS,YNS, VOL(25),VWD(25)
      COMMON /TIME/ T,DT,DTM,TFO,DTSN,TFND,DTRP,TRFP
      COMMON /CONS/ CU,SIA,SIR,CR,CT,ACCOM,PRES,CLWT(26),TCONS(25)
```

```
      COMMON /VARS/ XO,YO,XM,YM,UM,VM,WM,IUM
      COMMON /XCRD/ DRPR,DHPR,RPRFL(50),HPRFL(10)
      H=RR
      G=RI
      T=0.
C     CALCULATION OF GEOMETRICAL CONSTANTS
      YCR1=0.25*DIAMS
      YCR2=0.5*DIAMS
      DHPR=0.1*CR2
      YCR3=YCR2*THLIP
      YCR4=DIAMS
      YCR5=1.5*DIAMS
      YCR6=2.5*DIAMS
      DISMI=DISNS*XCR1
      SLI3=0.5*DIAMS/DISNS
      SLI4=0.5*SLI3
      SLI2=2.*SLI3
      SLI1=2.*SLIO
      SLO2=3.*SLO
      SLO3=YCR4/DISNS
      SLO4=YCR5/DISNS
      SLO5=YCR6/DISNS
      RADIN=YCR6*(1.05*XCR1)
      XCR5=(YCR4-YCR3)/(SLIO-SLIO5)
      DHPR=(CR1/5)/...
      YCR0=(YCR1-YCR3)/SLIO
      XCR1=(YCR2-YCR2)/SLI1
      HYP=SQRT(1.+SLO5*SL(05))
      XNS=SLO5/HYP
      YNS=...,/HYP
      HYP=SQRT(1.+SLO*SLO)
      XNO=SLO/HYP
      YNO=1./HYP
      HYP=SQRT(1.+SLI*SLI)
      XNI=SLI/HYP
      YNI=-1./HYP
C     CONSTANTS FOR INPUT DENSITY PROFILE
      ZNPR(1)=0.
      ZNPR(2)=SLI(4*XCR1*YCR)
      ZNPR(4)=ZNPR(2)
      ZNPR(4)=SLI1*XCR1*YCR2
      ZNPR(5)=ZNPR(4)
      ZNPR(6)=SLO3*XCR1*YCR4
      ZNPR(7)=ZNPR(6)
      ZNPR(8)=SLO4*XCR1*YCR5
      ZNPR(9)=ZNPR(8)
      ZNPR(10)=RADIN
      DIMENSION JWT(10)
      DATA JWT /1,1,2,2,11,11,13,13,16,16/
      DO 500 I=1,10
      CALL INPR(F,DF,ZNPR(I))
      J=JWT(I)
      FNPR(I)=F*CLWT(J)
  500 DFNPR(I)=DF*CLWT(J)
      FNPRM=(MAX(FNPR,10)
      DO 501 I=1,10
  501 FNPR(I)=FNPR(I)/FNPRM
C     VELOCITY COMPUTATION
      DIMENSION XVT(42),YVT(42)
      XVT(1)=XN1
      YVT(2)=XLM1
      XVT(3)=XLM1
      YVT(4)=XCM1
      XVT(5)=XCM1
      YVT(6)=XCM1
      XVT(7)=0
      XVT(8)=(YCR5-YCR4)/(SLO2-SLI4)
      YVT(9)=(YCR4-YCR3)/(SLI2-SLO3)
      XVT(11)=(YCR2-YCR3)/(SLIO-SLI3)
      YVT(11)=(YCR1-YCR5)/(SLO2-SLI4)
      XVT(12)=-YCR4/SLO2
      XVT(13)=0
      XVT(14)=0
      XVT(15)=0
      XVT(16)=0
      XVT(17)=0
      XVT(18)=(YCR5-YCR4)/(SLO1-SLO4)
      YVT(19)=(YCR4-YCR6)/(SLO1-SLO5)
      XVT(20)=(YCR2-YCR4)/SLO5
      YVT(21)=(YCR7-YCR3)/SLO
      YVT(22)=(YCR4-YCR5)/(SLO4-SLO)
      YVT(23)=(YCR4-YCR6)/(SLO3-SLO)
      XVT(24)=XCR2
      YVT(25)=XCR2
      YVT(26)=XCR2
      YVT(27)=XCR2
      YVT(28)=XCR2
      YVT(29)=XCR2
      YVT(30)=XCR3
      YVT(31)=XCR3
      YVT(32)=XCR3
      YVT(33)=XCR3
      YVT(34)=XCR3
      YVT(35)=XCR3
      YVT(36)=(YCR7-YCR3)/SLI
      YVT(37)=XCR4
      YVT(38)=XCR4
      YVT(39)=XCR4
      YVT(40)=XCR4
      YVT(41)=XCR4
      YVT(42)=XCR4
      YVT(1)=0
      YVT(1)=0
      YVT(2)=0
      YVT(3)=0
      YVT(4)=YCR5*SLI(4*XVT(2)
      YVT(6)=YCR4*SLI(4*XVT(1))
      YVT(8)=YCR4*SLI(4*YVT(4)
      YVT(25)=YCR4*SLI(4*YVT(25)
      YVT(36)=YCR4*SLI(4*YVT(36)
      YVT(41)=YCR4*SLI(4*YVT(4))
```

-26-

```
      YVT(5 )=YCR2+SI13=XVT(5 )
      YVT(10)=YCR2+SI13=XVT(10)
      YVT(15)=YCR2+SI13=XVT(15)
      YVT(26)=YCR2+SI13=XVT(26)
      YVT(33)=YCR2+SI13=XVT(33)
      YVT(40)=YCR2+SI13=XVT(40)
      YVT(4 )=YCR4+SIN3=XVT(4 )
      YVT(9 )=YCR4+SIN3=XVT(9 )
      YVT(17)=YCR4
      YVT(5 )=YCR5+SLN4=XVT(5 )
      YVT(8 )=YCR5+SLN4=XVT(8 )
      YVT(18)=YCR5+SLN4=XVT(18)
      YVT(37)=YCR7
      YVT(6 )=YCR6+SLN5=XVT(6 )
      YVT(7 )=YCR6+SLN5=XVT(7 )
      YVT(19)=YCR6+SLN5=XVT(19)
      YVT(20)=YCR6+SLN5=XVT(20)
      YVT(36)=YCR3+SLN6=XVT(36)
      YVT(25)=YCR3+SLN6=XVT(25)
      YVT(22)=YCR3+SLN6=XVT(22)
      YVT(21)=YCR3+SLN6=XVT(21)
      YVT(24)=YCR2+SI11=XVT(29)
      YVT(30)=YCR2+SI11=XVT(30)
      YVT(36)=YCR2+SI11=XVT(36)
      YVT(28)=YCR2+SI11=XVT(28)
      YVT(31)=YCR2+SI11=XVT(31)
      YVT(38)=YCR2+SI11=XVT(38)
      YVT(27)=YCR2+SI12=XVT(27)
      YVT(32)=YCR2+SI12=XVT(32)
      YVT(39)=YCR2+SI12=XVT(39)
      VOL(1 )=VPOL(XVT,YVT,4,12,11,2,1,0.,DISNS,VWD(1))
      VOL(2 )=VPOL(XVT,YVT,4,11,10,3,2,0.,DISNS,VWD(2))
      VOL(3 )=VPOL(XVT,YVT,4,13,14,11,12,0.,DISNS,VWD(3))
      VOL(4 )=VPOL(XVT,YVT,4,14,15,10,11,0.,DISNS,VWD(4))
      VOL(5 )=VPOL(XVT,YVT,4,24,25,14,13,0.,DISNS,VWD(5))
      VOL(6 )=VPOL(XVT,YVT,4,25,26,15,14,0.,DISNS,VWD(6))
      VOL(7 )=VPOL(XVT,YVT,4,35,34,25,24,0.,DISNS,VWD(7))
      VOL(8 )=VPOL(XVT,YVT,4,34,33,26,25,0.,DISNS,VWD(8))
      VOL(9 )=VPOL(XVT,YVT,4,42,41,34,35,0.,DISNS,VWD(9))
      VOL(10)=VPOL(XVT,YVT,4,41,40,33,34,0.,DISNS,VWD(10))
      VOL(11)=VPOL(XVT,YVT,4,10,9,4,3,0.,DISNS,VWD(11))
      VOL(12)=VPOL(XVT,YVT,5,15,16,23,9,10,0.,DISNS,VWD(12))
      VOL(13)=VPOL(XVT,YVT,4,9,8,5,4,0.,DISNS,VWD(13))
      VOL(14)=VPOL(XVT,YVT,4,17,18,9,8,0.,DISNS,VWD(14))
      VOL(15)=VPOL(XVT,YVT,4,21,22,18,17,0.,DISNS,VWD(15))
      VOL(16)=VPOL(XVT,YVT,4,8,7,6,5,0.,DISNS,VWD(16))
      VOL(17)=VPOL(XVT,YVT,4,8,18,19,7,0.,DISNS,VWD(17))
      VOL(18)=VPOL(XVT,YVT,5,22,21,20,19,18,DISNS,VWD(18))
      VOL(19)=VPOL(XVT,YVT,3,28,29,15,0.,0.,DISNS,VWD(19))
      VOL(20)=VPOL(XVT,YVT,4,33,32,27,26,0.,DISNS,VWD(20))
      VOL(21)=VPOL(XVT,YVT,4,40,39,32,33,0.,DISNS,VWD(21))
      VOL(22)=VPOL(XVT,YVT,4,32,31,28,27,0.,DISNS,VWD(22))
      VOL(23)=VPOL(XVT,YVT,4,39,38,31,32,0.,DISNS,VWD(23))
      VOL(24)=VPOL(XVT,YVT,4,31,30,29,28,0.,DISNS,VWD(24))
      VOL(25)=VPOL(XVT,YVT,5,38,37,36,30,31,DISNS,VWD(25))
C     END OF VOLUME COMPUTATION
      IVW=0.

      DO 1 I=1,18
    1 IVW=IVW+VWD(I)=CLMT(I)
      DO 2 I=1,25
    2 VWD(I)=VWD(I)=WST/IVW
      DO 201 I=1,NCL
  201 TCONS(I)=2.=FLOAT(WST)=VOL(I)=CLMT(I)/A/IVW
      CH=0.
      DO 202 I=1,5
      K=JWT(I=01)
  202 CH=CH+CLMT(K)=(YVT(I+1)==2-YVT(I)==2)
      CT=0.707=CR
      CH=(DISNS/DISNI)==2=3.14159=WST=CH/IVW
      SIA=0.707=SIA
      S=1./SIA
      DO 401 I=1,1475
  401 FH(I,1)=0.
      DO 402 I=1,60
  402 HPRE(I)=0.
      IF(NAPS.LT.0) RETURN
C     LOAD WST MOLECULES
      DIMENSION TFMS(25)
      DO 300 I=19,25
      TFMS(I)=CLMT(I)
  300 CLMT(I)=0.
      DO 102 I=1,NCL
  102 MAP(I)=0
      CLMTM=AMAX(CLMT,NCL)
      XCR41=XCR4-XCR1
      DCR4=DISNS=XCR4
      CDNS=DISNI=DCR4
      DO 30 I=1,WST
      XN=CDNS/(DCR4-XCR1=RANDL(R))-DISNS
      YN=YCR7=SQRT(RANDL(R))
      LP=LOC(XN,YN)
      IF(CLMT(LP)/CLMTM-RANDL(R))110,10,20
   20 XN=XP=DISNS
      HYP=SQRT(YN=YN+XN=XN)
      UR=XN/HYP=SIR=RANDG(G)
      VR=YN/HYP=SIR=RANDG(G)
      WR=SIR=RANDG(G)
      IDN=0
   30 CALL FIT(LP,E40)
      GO TO 40
   40 WRITE(6,50)
   50 FORMAT(18H OVERFLOW, SETUP)
   60 CONTINUE
      DO 301 I=19,25
  301 CLMT(I)=TFMS(I)
      RETURN
      END
      FUNCTION VPOL(X,Y,NV,N1,N2,N3,N4,N5,C,FN)
      DIMENSION N(5),X(1),Y(1)
      FUN(Z)=ALOG(1.+(S+N/Z)==2)
      N(1)=N1
      N(2)=N2
      N(3)=N3
      N(4)=N4
      N(5)=N5
      F=0.
      V=0
      DO 1 I=1,NV
      NI=N(I)
      J=MOD(I,NV)+1
      NJ=N(J)
      A=C+X(NI)
      B=C+X(NJ)
      DX=B-A
      IF(ABS(DX).LT.1.E-3) GO TO 1
      S=(Y(NJ)-Y(NI))/DX
      D=(B=Y(NI)-A=Y(NJ))/DX
      DX=DX/10.
      FI=(FUN(A)+FUN(B))/2.
      DO 2 K=1,9
    2 FI=FI+FUN(A+K=DX)
      F=F-FI=DX
    1 V=V+1.04720=(Y(NI)==2+Y(NI)=Y(NJ)+Y(NJ)==2)=(X(NI)-X(NI))
C     1.04720=PI/3
      FN=3.14159=C=C=F
      VPOL=V
      RETURN
      END
      SUBROUTINE INPR(F,DF,R)
C     SKIMMER 1970
      COMMON /GEOM/ PFI,DISNS,DIAMS,THLIP,SLO,SLI,XCR1,XCR2,XCR3,XCR4,
     X     SLI1,SLI2,XCR0,XCR1,XCR5,YCR1,YCR2,YCR3,YCR4,YCR6,YCR7,
     X SLO1,SLO2,SLO3,SLO4,SLO5, SLI3,SLI4, RADIN,DISNI, XN1,YN1,
     X XN0,YN0, XN5,YN5, VOL(25)
      A=ATAN(R/DISNI)
      A2=2.=A
      AP=PFI=A
      WA=1.+COS(A2)
      WB=1.+COS(AP)
      W=WA=WB
      F=R=W
      DA=DISNI/(DISNI=DISNI+R=R)
      DF=W-(2.=R=SIN(A2)=WB+PFI=R=WA=SIN(AP))=DA
      RETURN
      END
      FUNCTION LOC(X,Y)
C     SKIMMER 1970
      INTEGER TA,TB,TC,TD,TE,TF,TG,TH,TI,TJ,TK,TL, TM,TN,TO,TP
      COMMON /GEOM/ PFI,DISNS,DIAMS,THLIP,SLO,SLI,XCR1,XCR2,XCR3,XCR4,
     X     SLI1,SLI2,XCR0,XCR1,XCR5,YCR1,YCR2,YCR3,YCR4,YCR6,YCR7,
     X SLO1,SLO2,SLO3,SLO4,SLO5, SLI3,SLI4, RADIN,DISNI, XN1,YN1,
     X XN0,YN0, XN5,YN5, VOL(25)
      LOGICAL LIDFR(25),LMASK(25),LIX,LIXM
      DIMENSION IDFR(25),MASK(25)
      EQUIVALENCE (LIDFR(1),IDFR(1)),(LMASK(1),MASK(1)),
     X (LIX,IX),(LIXM,IXM)
      DATA MASK/Z10021,Z10073,Z8021,Z8023,ZC001,ZC003,Z6001,Z6003,Z3001,
     X Z3003,Z10026,Z80A6,Z1002C,ZAC,ZCC,Z1U03A,Z78,Z20A,Z4102,
     X Z8A02,Z3802,Z6C00,Z3C00,Z6600,Z3700/,
     X IDFR/ Z10001,Z10002,Z21,Z22,ZC001,ZC002,Z7001,Z2002,Z1001,Z1002,
     X Z10004,ZA4,Z1000A,Z2A,ZCR,Z10010,Z30,Z2D0,Z4100,
     X Z2800,Z1800,Z2400,Z1400,Z7100,Z1300/,
     X TA/27/,TB/24/,TC/2A/,TD/210/,TE/220/,TF/240/,TG/280/,
     X TH/2100/,TI/2200/,TJ/2400/,TK/2800/,TL/71000/,TM/22000/,
     X TN/24000/,TO/28000/,TP/210000/
    3 IX= ISGN(SLI4=X+YCH1-Y) + TA=ISGN(SLI3=X+YCR2-Y)
     X + TB=ISGN(SLO3=X+YCR4-Y) + TC=ISGN(SLN4=X+YCR5-Y)
     X + TD=ISGN(SLO5=X+YCR6-Y) + TE=ISGN(SLI2=X+YCR6-Y)
     X + TF=ISGN(SLO1=X+YCR6-Y) + TG=ISGN(Y-YCR3-SLO=X)
     X + TH=ISGN(SLI6=X+YCR7-Y) + TI=ISGN(YCR7-Y)
     X + TJ=ISGN(SLI1=X+YCR2-Y) + TK=ISGN(SLI2=X+YCR2-Y)
     X + TL=ISGN(XCR4-X) + TM=ISGN(XCR3-X) + TN=ISGN(XCR2-X)
     X + TO=ISGN(X) + TP=ISGN(X-XCR1)
      DO 1 I=1,25
      LIXM=LIX.AND.LMASK(I)
      II=I
      IF (IXM-IDFR(I)) 1,2,1
    1 CONTINUE
      LOC=26
      RETURN
    2 LOC=II
      RETURN
      END
      SUBROUTINE COLI(I)
C     SKIMMER 1970
      COMMON /STORE/ NCL,NCP,MAP(30),X(5000),Y(5000),U(5000),V(5000),
     X W(5000),ID(5000)
      COMMON /CONS/ CH,SIA,SIR,CR,CT,ACCOM,PRES,CLMT(26),TCONS(25)
      COMMON /RAND/R,G
      COMMON /TIME/ T,DT,DTM,TEO,DTSW,TEND,DTRP,TREP
      NAMELIST/CHEK/ KA,KR,WA,WR,ITFRY,UA,VA,WA,UR,VR,WR,MAP
      DOT=0.
      IF (I-1) 1,1,2
    1 KA=0
      GO TO 3
    2 KA=MAP(I-1)
    3 KR=MAP(I)
      FKRA=FLOAT(KR-KA)
      KA=KA+1
      IF (KR-KA) 10,10,4
    4 CCT=TCONS(I)/FKRA/FKRA
      VMAX=SQRT(SOM(KA,KR,U)+SOM(KA,KR,V)+SOM(KA,KR,W))
      IF(VMAX) 10,10,5
    5 MA=KA+IFIX(FKRA=W=RANDL(R))
      MR=KA+IFIX(FKRA=RANDL(R))
      IF (MR-MA) 6,5,6
    6 UA=U(MA)
      VA=V(MA)
      WA=W(MA)
      UR=U(MR)
      VR=V(MR)
      WR=W(MR)
      URA=UR-UA
      VRA=VR-VA
      WRA=WR-WA
      VREL=SQRT(URA=URA+VRA=VRA+WRA=WRA)
      IF(VREL.LT.VMAX=RANDL(R)) GO TO 5
      TCC=CCT/VREL
      DOT=DOT+TCC
```

```
      IF(DDT.LT.DTM) GO TO 7
      IF(RAND1(N).GT.(DDT-DTM)/TCH) GO TO 7
      DDT=DDT-TC3
      GO TO 10
    7 FU=WANDV(4)
      FV=WANDV(U)
      FW=WANDV(N)
      F=SQNT(FU*FU+FV*FV+FW*FW)
      IF (F) 7,7,8
    8 IF (F-1.) 9,9,7
    9 FU=VRF1*FU/F
      FV=VRF1*FV/F
      FW=VRF1*FW/F
      U(MA)=0.5*(UA+UA-FU )
      V(MA)=0.5*(VA+VA-FV)
      W(MA)=0.5*(WA+WA-FW)
      ID(MA)=ID(MA)+2
      U(MR)=0.5*(UA+UA-FU)
      V(MR)=0.5*(VA+VA-FV)
      W(MR)=0.5*(WA+WA-FW)
      ID(MR)=ID(MR)+2
      ITFRY=I
      IF(VRF1.EQ.0.) WRITE(6,CHEK)
      IF (DD.-DTM) 5,10,10
   10 DT=DT+DDT*FKRA
      RETURN
      END
      FUNCTION SOM(L,M,X)
C     SKIMMER 1970
      DIMENSION X(1)
      A=X(1)
      B=A
      DO 4 I=1,M
      XM=X(I)
      IF (XM-A) 1,2,2
    1 A=XM
    2 IF (XM-B) 4,4,3
    3 B=XM
    4 CONTINUE
      SOM=(B-A)*(B-A)
      RETURN
      END
      SUBROUTINE TRRKP(MARS)
C     SKIMMER 1970
      COMMON /TIME/ T,DT,DTM,TFO,DTSM,TEND,DTRP,TRFP
      COMMON /STORE/ NCL,MCP,MAP(30),X(5000),Y(5000),U(5000),V(5000),
     X W(5000),ID(5000)
      COMMON /CONS/ CU,S1A,S1R,CR,CT,ACCOM,PRES,CLWT(26),TCONS(25)
      COMMON /VARS/ X0,Y0,XM,YM,UM,VM,WM,IDM
      DATA IC/16384/
      M=1
    1 MPCL=MAP(NCL)
    2 IF (M-MPCL) 4,4,3
    3 RETURN
    4 IDM=ID(M)
      IF (IDM/IC) 6,6,5
    5 ID(M)=IDM-IC
      M=M+1

      GO TO 2
    6 LO=INVRS(MAP,NCL,M)
      X0=X(M)
      Y0=Y(M)
      UM=U(M)
      VM=V(M)
      WM=W(M)
      CALL FRP(MARS)
   26 LM=LOC(XM,YM)
      CLO=CLWT(IM)/CLWT(IO)
      ICLO=INTR(CLO)
      IF (ICLO) 7,7,14
   14 IF (LM-LO) 9,12,10
    9 CALL DOWN(MAP,M,LO-1+LM)
      M=M+ICLO
      GO TO 11
   10 CALL UP(MAP,M,LO,LM)
      IDM=IDM+IC
   11 MM=MAP(LM)
      GO TO 13
   12 MM=M
      M=M+1
   13 CALL TRS(MM)
      IF(ICLO-1) 1,1,15
   15 DO 151 I=2,ICLO
  151 CALL FIT(IM,G1)
      GO TO 1
    7 CALL UP(MAP,M,M,NCL)
      MAP(NCL)=MPCL-1
      GO TO 1
      END
      FUNCTION INVRS(MAP,I,M)
      DIMENSION MAP(1)
      I=0
      J=L
    9 IF (J-I-1) 10,10,6
   10 INVRS=I
      RETURN
    6 K=(I+J)/2
      IF (M-MAP(K)) 7,7,8
    7 J=K
      GO TO 9
    8 I=K
      GO TO 9
      END
      SUBROUTINE UP(MAP,M,L,K)
      DIMENSION MAP(1)
      LX=L
      MA=M
    1 MR=MAP(LX)
      CALL TR(MA,MR)
      IF (LX-K) 3,2,2
    2 RETURN
    3 MAP(LX)=MR-1
      LX=LX+1
      MA=MR
      GO TO 1

      END
      SUBROUTINE DOWN(MAP,L,I,K)
      DIMENSION MAP(1)
      LX=L
      MA=M
    1 MR=MAP(LX)+1
      CALL TR(MA,MR)
      MAP(LX)=MM
      IF (LX-K) 2,2,3
    2 RETURN
    3 LX=LX-1
      MA=MR
      GO TO 1
      END
      SUBROUTINE TRS(M)
      COMMON /VARS/ X0,Y0,XM,YM,UM,VM,WM,IDM
      COMMON /STORE/ NCL,MCP,MAP(30),X(5000),Y(5000),U(5000),V(5000),
     X W(5000),ID(5000)
      X(M)=XM
      Y(M)=YM
      U(M)=UM
      V(M)=VM
      W(M)=WM
      ID(M)=IDM
      RETURN
      END
      SUBROUTINE TR(M,N)
      COMMON /STORE/ NCL,MCP,MAP(30),X(5000),Y(5000),U(5000),V(5000),
     X W(5000),ID(5000)
      X(M)=X(N)
      Y(M)=Y(N)
      U(M)=U(N)
      V(M)=V(N)
      W(M)=W(N)
      ID(M)=ID(N)
      RETURN
      END
      SUBROUTINE FRP(MARS)
C     SKIMMER 1970
      COMMON /GEOM/ PF1,DISNS,DIANS,THI,IP,SLO,SI1,XCR1,XCR2,XCR3,XCR4,
     X  SI11,SL12,XCR0,XCR1,XCR5,YCR1,YCR2,YCR3,YCR4,YCR5,YCR6,YCR7,
     X  SLO1,SL02,SL03,SL04,SL05, SL13,SL14, RADIN,DISNI, XNI,YNI,
     X  XN0,YN0, XN5,YN5, VOI(26)
      COMMON /VARS/ X0,Y0,XM,YM,UM,VM,WM,IDM
      COMMON /CONS/ CU,S1A,S1R,CR,CT,ACCOM,PRES,CLWT(26),TCONS(25)
      COMMON /HAND/ R,G
      COMMON /NCRD/ DRPR,DHPR,RPRFI(50),MPRFI(10)
      COMMON /TIME/ T,DT,DTM,TFO,DTSM,TEND,DTRP,TRFP
      COMMON /SW/ KSAM,KRFP
      TL=DT
   31 CALL LCHK(SL05,YCR6,XCR1,XCR5,-1.,R5,IL)
      CALL LCHK(0.,YCR7,XCR5,XCR0,-1.,F7,TL)
   32 CALL LCHK(SL0,YCR3,0.,XCR0,1.,G9,TL)
      CALL LCHK(SL11,YCR2,0.,XCR1,-1.,G12,TL)
C     CHECK LIP
      IF(IDM) 19,19,1
    1 IF(X0) 2,2,21
    2 TT=-X0/UM

      IF(TL-TT) 19,3,3
    3 CALL TRANS(TT,0)
      IF(YCR2-YM) 4,4,1M
    4 IF(YCR3-YM) 19,15,15
C     REAM PROFILE
   21 TT=(XCR4-X0)/UM
      IF(TL-TT) 19,22,22
   22 GO TO(19,23),KSAM
   23 CALL TRANS(TT,0)
      CALL SIFT(0.,DRPK,RPRFI,50,1,.YM)
      GO TO 19
    5 CALL REFL(XN5,YN5)
      UM=UM-PRES*XN5
      VM=VM-PRES*YN5
      GO TO 32
    7 CALL REFL(0.,-1.)
      VM=VM+PRES
      GO TO 32
    9 IF(ACCOM-RANDL(R)) 10,10,11
   10 CALL REFL(XN0,YN0)
      IF(MARS.LT.0) YO=10000.
      GO TO 31
   11 CALL FMIT(XN0,YN0)
      IF(MARS.LT.0) YO=10000.
      GO TO 31
   12 IF(ACCOM-RANDL(R)) 13,13,14
   13 CALL REFL(XNI,YNI)
      IF(MARS.LT.0) YO=10000.
      GO TO 19
   14 CALL FMIT(XNI,YNI)
      IF(MARS.LT.0) YO=10000.
      GO TO 19
   15 TI=TL-TT
      IF (ACCOM-RANDL(R)) 16,16,17
   16 CALL TRANS(TT,1)
      CALL REFL(-1.,0.)
      IF(MARS.LT.0) YO=10000.
      GO TO 31
   17 CALL FMIT(-1.,0.)
      IF(MARS.LT.0) YO=10000.
      GO TO 31
   18 GO TO (19,24), KSAM
   24 CALL SIFT(0.,DHPR,MPRFI,10,1,.YM)
   19 CALL TRANS(TL,1)
      RETURN
      END
      SUBROUTINE TRANS(DT,III)
      COMMON /VARS/ X,R,XNF,RNEW,VX,VR,VC
      VVR=VR
      VVO=VO
      TVO=DT*VO
      RM=R+DT*VR
      XNEW=X+DT*VX
      RNEW=SQRT(RM*RM+TVO*TVO)
      IF (RNEW) 1,2,1
    1 IF(III.EQ.0) RETURN
      SN=TVO/RNEW
```

```
        CS=VM/VREL
        VM=SM+VV(1)+CS*VR
        VI=SQ*VV(1)-SM*VVR
        RETURN
2       VM=V1
        RETURN
        END
        SUBROUTINE LCHK(S,R,X1,X0,X,P,TLEFT)
        COMMON /VARS/ X0,Y0,XM,YM,UM,VM,SM,IUM
C       THIS SUBROUTINE CHECKS WHETHER A MOLECULE COLLIDES WITH THE SEGMENT
C       FROM X0 TO X0 OF THE LINE Y=S*X+R . (1 IS A PARAMETER DESIGNATING)
C       SIDE IF LINE FROM WHICH COLLISION CAN OCCUR
        D1=(1.+1
        TH/X=H,
        TI=R,
        AI=X0+S*X+R
        IF(VM-S*UM)*X1 2,17,17
2       IF((Y0-AI)*X1) 17,16,3
3       C1=VM*VM+2.*SM*UM -(S*UM)**2
        C2=Y0*VM-AI*S*UM
        C3=Y0*Y0-AI*AI
        IF(C1) 7,6,7
4       IF(C2) 6,17,6
6       TT=-0.5*C3/C2
        GO TO 16
7       C2=C2/C1
        C3=C3/C1
        DISCR=C2*C2-C3
        IF(DISCR) 17,9,9
9       RTDIS=SQRT(DISCR)
        TMAX=RTDIS-C2
        TMIN=-RTDIS-C2
        IF (TMAX.LT.0) GO TO 17
        IF (TMIN.LT.0) GO TO 12
        TT=TMIN
        GO TO 16
12      TT=TMAX
14      IF(DT-TT) 17,15,15
15      CALL TRANS(TT,0)
        IF(X1.LT.XM .AND. XM.LT.X0) GO TO 16
        IF (TT.LT.TMAX) GO TO 12
17      RETURN
16      TLEFT=DT-TT
        CALL TRANS(TT,1)
        RETURN1
        END
        SUBROUTINE EMIT(C,S)
C       SKINNER 1970
        COMMON /CONS/ CU,STA,SIR,CR,CT,ACCOM,PRES,CLHT(25),TCONS(25)
        COMMON /VARS/ X0,Y0,XM,YM,UM,VM,SM,IUM
        DATA IC/163R6/
        COMMON /RAND/ R,G
        VN=CT*SQRT(-ALOG(RANDU(R)))
        VI=CR*RANDG(G)
        UM=C*VN-S*VI
        VM=C*VI+S*VN
        WM=CR*RANDG(G)
        X0=XM
        Y0=YM
        IUM=IC*(IUM/IC)
        RETURN
        END
        SUBROUTINE REFL(XM,YM)
        COMMON /VARS/ X0,Y0,XM,YM,UM,VM,SM,IUM
        DATA IC/163R6/
        VN=UM*2.*(UM*XM+VM*YM)
        UM=UM-VM*XM*XM
        VM=VM-VM*XM*YM
        X0=XM
        Y0=YM
        IUM=IC*(IUM/IC)
        RETURN
        END
        SUBROUTINE INPUT
C       SKINNER 1970
        INTEGER R,G
        COMMON /GEOM/ PFI,DISNS,DIANS,THIIP,SLO,SI1,XCR1,XCR2,XCR3,XCR4,
     X       SI11,SI12,XCR0,XCR1,XCR5,YCR1,YCR2,YCR3,YCR4,YCR5,YCR6,YCR7,
     X   SLO1,SLO2,SLO3,SLO4,SLO5, SLE3,SLE4, RADIN,DISN1, XM1,YM1,
     X   ZN1,YN0, ZN5,YN5, VN1(25)
        COMMON /VARS/ X0,Y0,XM,YM,UM,VM,SM,IUM
        COMMON /CONS/ CU,STA,SIR,CR,CT,ACCOM,PRES,CLHT(25),TCONS(25)
        COMMON /NPREL/ ZNPR(10),ENPR(10),DENPR(10)
        COMMON /TIME/ T,DT,DTM,TFO,DTSM,TEND,DTKP,TREP
        COMMON /RAND/ R,G
        NIM=INTR(CNOOT)
        DO 6 I=1,NIM
M18     X0=XCR1
1       Y0=RADIN*RANDU(R)
        IF((ASPL(ZNPR,ENPR,DENPR,10,Y0)-RANDU(R)) 1,5,4
4       HYP=SQRT(DISN1*DISN1+Y0*Y0)
        VX1=DISN1/HYP
        VK1=Y0/HYP
N       Z=VX1*SI4*RANDG(G)
5       IF(Z-1.5*RANDU(R)) N,5,5
5       IN=Z
        VM=VK1+SI4*RANDG(G)
        WM=SI4*RANDG(G)
        IUM=0
        CALL TRANS(DT*RANDU(R))
        TI,1=UC(XM,YM)
        IF((TI,1.LT.25) GO TO 6
        WRITE(6,914)
914     FORMAT(1H ,*ERROR IN LOCI)
        GO TO FIN
6       CALL FIT(TI,I,57)
7       RETURN
        END
        FUNCTION CASPL(Z,F,DF,T,X)
C       SKINNER 1970
C       SUPPOSE F IS A FUNCTION WE WISH TO COMPUTE, AND WE ARE GIVEN A VECT
C       OR OF DIMENSION T OF ARGUMENT POINTS, A VECTOR G CONTAINING THE CORR
C       F(I)=F(I) AND A VECTOR H CONTAINING THE FUNCTION 3RD I=H*I),
C       CASPL COMPUTES AN INTERPOLATED VALUE OF F AT X.
        DIMENSION Z(1),F(1),DF(1)
```

```
C       BINARY SEARCH FOR INTERVAL Z(I),Z(J) (J=I+1) WHERE X OCCURS
        I=1
        J=N
A       K=(I+J)/2
        IF(X-Z(K)) 7,7,R
7       J=K
        GO TO 9
8       I=K
9       IF(J-I-1) 10,10,A
C       END OF BINARY SEARCH
        Z1=Z(I)
        ZJ=Z(J)
        ZIJ=ZJ-Z1
        X1=(X-Z1)/ZIJ
        XJ=(ZJ-X)/ZIJ
        FI=F(I)
        FJ=F(J)
        DFI=Z1*DF(I)
        DFJ=1,J*DF(J)
        GI=DFI-2.*FI
        GJ=DFJ-2.*FJ
        HI=3.*FI-DFI
        HJ=3.*FJ-DFJ
        CASPL=X1*(XI*(XJ*XJ*(GI+XJ*HI))+X1*FI)+XJ*XJ
        RETURN
        END
        SUBROUTINE TRAN
        COMMON /STORE/ NCL,NCP,MAP(30),X(5000),Y(5000),U(5000),V(5000),
     X   W(5000),ID(5000)
        COMMON /VARS/ X0,Y0,XM,YM,UM,VM,WM,IUM
        COMMON /TIME/ T,DT,DTM,TFO,DTSM,TEND,DTKP,TREP
        COMMON /HIST/ FU(10,25),FV(10,25),FW(10,25),CFU(10,25),    -25),
     X   CFW(10,25),UR(25),VR(25),WR(25),UUR(25),VVR(25),WWR(    .25)
        MLAST=0
        DO 6 I=1,NC,
        MFIRST=MLAST+1
        MLAST=MAP(I)
        LSI/=MLAST+1-MFIRST
        IF((LSI/E) 6,6,7
7       D=DT/FLOAT(LSI/E)
        DO K=MFIRST,MLAST
        IUM=ID(K)
        VM=V(K)
        VM=W(K)
        MTAG=MOD(ID(K),32)/2
        IF (MTAG) 1,1,2
        CALL SIFT(-1.5,0.3,FU(1,I),10,DT,UM)
        CALL SIFT(-1.5,0.3,FV(1,I),10,DT,VM)
        CALL SIFT(-1.5,0.3,FW(1,I),10,DT,WM)
        GO TO 5
2       IF(MTAG/2) 4,4,3
3       CINCL,J=CINCL(I)+DT
        CALL SIFT(-1.5,0.3,CFU(1,I),10,DT,UM)
        CALL SIFT(-1.5,0.3,CFV(1,I),10,DT,VM)
        CALL SIFT(-1.5,0.3,CFW(1,I),10,DT,WM)
        UR(I)=UR(I)+UM*D
        VR(I)=VM(I)+VM*D
        WR(I)=WR(I)+WM*D
        UUR(I)=UUR(I)+UM*VM*D
        VVR(I)=VVR(I)+VM*VM*D
        WWR(I)=WWR(I)+WM*WM*D
R       CONTINUE
        RETURN
        END
        SUBROUTINE FIT(I,C)
        COMMON /VARS/ X0,Y0,XM,YM,UM,VM,WM,IUM
        COMMON /TIME/ T,DT,DTM,TFO,DTSM,TEND,DTKP,TREP
        COMMON /STORE/ NCL,NCP,MAP(30),X(5000),Y(5000),U(5000),V(5000),
     X   W(5000),ID(5000)
        NCP=MAP(NCL)+1
        IF (NC-NCP) 3,3,1
1       WRITE (6,2)
2       FORMAT (17H STORAGE OVERFLOW)
        TEND=T
        RETURN 1
3       CALL TRAN (MAP,NC,NCL,L)
        N=MAP(L)
        CALL INS(N)
        RETURN
        END
        SUBROUTINE SIFT(A,D,X,L,T,XI)
C       TO ACCUMULATE A HISTOGRAM FOR THE VARIABLE W WHERE X IS THE
C       HISTOGRAM ARRAY OF DIMENSION N, IF A+(K-1)D .LT.W .LT. A+KD,
C       T IS ADDED TO X(K), IF W FALLS OUTSIDE ARRAY RANGE IT CONTRIBUTES
C       TO THE NEAREST BUCKET.
        DIMENSION X(N)
        K=(E)X((X-A)/D)+1
        IF(K) 2,2,1
1       IF (N-K) 3,4,4
3       K=1
        GO TO 4
2       K=N
4       X(K)=X(K)+T
        RETURN
        END
        FUNCTION INTR(X)
        COMMON /RAND/ R
        I=INT(X)
        IF (X-FLOAT(I)-RANDU(R)) 2,1,1
1       I=I+1
2       INTR=I
        RETURN
        END
        FUNCTION RANDU(IX)
        A=0.0
        DO 50 I=1,12
50      A=A+RANDU(IX)
        RANDG=A-6.0
        RETURN
        END
        FUNCTION RANDU(IX)
        IY=IX*65539
        IF(IY) 5,6,6
5       IY=IY+2147483647+1
```

```
6    YFL=IY
     YFL=YFL*.4656613 F=9
     IX=IY
     RANDL=YFL
     RETURN
     END
     FUNCTION RANDR(X)
     RANDR   =2.*RANDL(Y)-1.
     RETURN
     END
     FUNCTION UMAX(X,L)
     DIMENSION X(1)
     Y=X(1)
     DO 1 I=2,5
     IF(Y-X(1)) 2,1,1
2    Y=X(1)
1    CONTINUE
     UMAX=Y
     RETURN
     END
     FUNCTION ISGN(X)
     ISGN=0
     IF(A.GF.0.) ISGN=1
     RETURN
     END
     SUBROUTINE OUR?
     COMMON /STORF/ NCL,NCP,P(120,50)
     COMMON /RCRD/ DRPR,DHPR,RPRFL(50),HPRFL(10)
     COMMON /GEOM/ PFI,DISNS,DIAMS,THLIP,SLO,SLI,XCR1,XCR2,XCR3,XCR4,
    X     SLI1,SLI2,XCR0,XCR1,XCR5,YCR1,YCR2,YCR3,YCR4,YCR5,YCR6,YCR7,
    X XNI,YNO, XN5,YN5, VOL(25),VHD(25)
     COMMON /CONS/ CU,SIA,SIR,CR,CT,ACCON,PRES,CLMT(26),TCONS(25)
     COMMON /REGIN/ RR,RG,A,NST
     COMMON /TIME/ T,DT,DTM,TFO,DTSM,TEND,DTRP,TRFP
     COMMON /HIST/ FU(10,25),FV(10,25),FW(10,25),CFU(10,25),CFV(10,25),
    1CFW(10,25),UR(25),VR(25),WR(25),UUR(25),VVR(25),WWR(25),CORG(25)
     COMMON /RAND/R,G
     INTEGER RR,RG ,R,G
     NAMELIST/GEO/ PFI,DISNS,DIAMS,THLIP,SLO,SLI,XCR1,XCR2,XCR3,XCR4,
    X     SLI1,SLI2,XCR0,XCR1,XCR5,YCR1,YCR2,YCR3,YCR4,YCR5,YCR6,YCR7,
    X SLI1,SLI2,SLO,,SLO4,SLO5, SLI3,SLI4, RADIN,DISNI, XNI,YNI,
    X XNI,YNO, XN5,YN5, VOL
     NAMELIST/CON/ CU,SIA,SIR,CR,CT,ACCON,PRES,CLMT     ,TCONS
     NAMELIST/REG/ RR,RG,A,NST,R,G
     NAMELIST/TIM/ T,DT,DTM,TFO,DTSM,TEND,DTRP,TRFP
     WRITE(6,1)
1    FORMAT('1',49X,'SKIMMER 1971'///)
     WRITE(6,GEO)
     WRITE(6,CON)
     WRITE(6,REG)
     WRITE(6,TIM)
     FLT=T-TFO
C PREPARATION OF HISTOGRAMS FOR OUTPUT
     THPR=0.
     DO 21 I=1,50
     I=LOC(XCR4,I*DRPR)
     IF(I.LT.11) TRPR=RPRFL(I)/FLT+TRPR

21   RPRFL(I)=RPRFL(I)/(CLMT(L)*FLT*6.283)*R*(I-0.5)*DRPR)
     THPR=0
     DO 22 I=1,10
     I=LOC(0,I*DHPR)
     THPR=HPRFL(I)/CLMT(L)/FLT
22   HPRFL(I)=HPRFL(I)/(CLMT(L)*FLT*6.283)*R*(I-0.5)*DHPR)
     TNI=1.570794*YCR2*YCR2*A*TCONS(I)/VOL(I)/CLMT(I)
     DO 24 I=1,NCL
     FTWT=FLT*CLMT(L)
     DO 23 I=1,10
     FU(I,L)=FU(I,L)/FTWT
     FV(I,L)=FV(I,L)/FTWT
     FW(I,L)=FW(I,L)/FTWT
     CFU(I,I)=CFU(I,L)/FTWT
     CFV(I,I)=CFV(I,L)/FTWT
23   CFW(I,I)=CFW(I,L)/FTWT
     CORG(I)=CORG(I)/FTWT
     UR(L)=UR(L)/FLT
     VR(L)=VR(L)/FLT
     WR(L)=WR(L)/FLT
     UUR(L)=UUR(L)/FLT
     VVR(L)=VVR(L)/FLT
24   WWR(L)=WWR(L)/FLT
C PRINTING OF HISTOGRAMS
     WRITE(6,41)
41   FORMAT(///,30X,'FLUX PROFILE AT ORIFICE',//,' DISTANCE FROM AXIS',
    X  45X, 'NUMBER PER UNIT TIME',//)
     CALL HIST1(HPRFL,10,P,2,40,TOTH)
     DO 42 J=1,10
42   P(1,J)=J*DHPR
     WRITE(6,43)  ((P(I,J),I=1,42),J=1,10)
43   FORMAT(10(F10.5,5X,40A),F10.5/))
     WRITE(6,44)       TOTH,THPR,TNI
44   FORMAT(' TOTAL',48X,F10.5//,' NUMBER/TIME THRU ORIFICE =',F10.5,
    X ',  THEORETICAL FIGURE FOR NO SKIMMER INTERACTION =',F10.5)
     WRITE(6,45)
45   FORMAT(1H1,36X,'BEAM FLUX PROFILE AT END OF FLOWFIELD',//,
    X ' DISTANCE FROM AXIS',40X,'NUMBER PER UNIT TIME',//)
     CALL HIST1(RPRFL,50,P,2,100,TOTR)
     DO 46      J=1,50
46   P(1,J)=J*DRPR
     WRITE(6,47) ((P(I,J),I=1,102),J=1,50)
47   FORMAT(50(F10.5 ,100A1,F10.5/))
     WRITE(6,48) TRPR,TOTR
48   FORMAT(/,' NUMBER/TIME IN BEAM',F10.5,75X,'TOTAL',F10.5)
C DISTRIBUTION FUNCTION PART
     DO 72 L=1,NCL
     WRITE(6,6) L
6    FORMAT
    X (1H1,45X,'VELOCITY HISTOGRAMS FOR CELL ',I4,' THE FIRST HISTOGR
    XAM TRIPLE REFERS TO MOLECULES WHICH HAVE UNDERGONE NO COLLISIONS I
    XN THE FLOWFIELD,'/        ' THE SECOND TO MOLECULES WHICH HAVE UNDE
    XRGONE AT LEAST ONE COLLISION, AND THE THIRD GIVES THE SUM OF THE F
    XIRST TWO,'/          ' LOWER CELL BOUNDARY VALUES APPEAR DOWN THE
    XLEFT, TIME AVERAGED NUMBER OF MOLECULES APPEARS AT THE RIGHT OF EA
    XCH BAR,'///)
     CALL HIST3(P,FU,FV,FW,L)

     CALL HIST3(P,CFU,CFV,CFW,L)
     DO 71 I=1,10
     FU(I,L) = FU(I,L)+CFU(I,L)
     FV(I,L) = FV(I,L)+CFV(I,L)
71   FW(I,L) = FW(I,L)+CFW(I,L)
     CALL HIST3(P,FU,FV,FW,L)
     WRITE(6,74) VVR(L)
72   WRITE(6,73) ... ,VVR(L),WWR(L),CORG(I)
73   FORMAT(' MEAN VELOCITY VECTOR',3F10.5,'     MEAN SQUARE VELOCITY VE
    XCTOR',3F10.5/,          ' TIME AVERAGED NUMBER OF MOLECULES WHICH HAV
    XE UNDERGONE MORE THAN ONE COLLISION IN THE FLOWFIELD ',F10.5)
74   FORMAT( /,' THEORETICAL NO INTERACTION TOTAL =',F10.5//)
     WRITE(6,1)
     RETURN
     END
     SUBROUTINE BAR(P,L,X)
     DIMENSION P(1)
     DATA STAR,BLANK/4H* ,4H  /
     N=IFIX((FLOAT(L)-1.F-A)*X)+1
     DO 1 I=1,N
1    P(I)=STAR
     N=N+1
     DO 2 I=N,L
2    P(I)=BLANK
     RETURN
     END
     SUBROUTINE HIST1(F,N,P,K,L,T)
C F IS THE N-VECTOR WHERE THE FIGURES ARE STORED
C P IS A 120XN ARRAY WHERE THE HISTOGRAM BARS WILL BE SET UP
C K IS THE INITIAL CHAR LOCATION OF EACH BAR
C L IS THE LENGTH OF THE BARS
C T IS THE LOCATION WHERE THE TOTAL WILL BE STORED
     DIMENSION P(120,1),F(1)
     U=UMAX(F,N)
     IF(U) 2,3,2
3    U=1.
2    S=0
     KL=K+L
     DO 1 I=1,N
     Z=F(I)
     S=S+Z
     CALL BAR(P(K,I),L,Z/U)
1    P(KL,I)=Z
     T=S
     RETURN
     END
     SUBROUTINE HIST4(P,A,B,C,L)
     DIMENSION P(120,1), A(10,1),B(10,1),C(10,1)
     DO 4 J=1,10
4    P(1,J)=-1.8+0.3*FLOAT(J)
     CALL HIST1(A(1,L),10,P,2,20,TA)
     CALL HIST1(A(1,L),10,P,23,20,TB)
     CALL HIST1(C(1,L),10,P,44,20,TC)
     WRITE(6,1)  ((P(I,J),I=1,64),J=1,10)
1    FORMAT(10(F10.5,5X,20A1,F10.5,5X,20A1,F10.5,5X,20A1,F10.5/)
     WRITE(6,2) TA,TB,TC
2    FORMAT(7H TOTA S,29X,F10.5,25X,F10.5,25X,F10.5/ )
     RETURN
     END
```

APPENDIX 2

Typical Output

<u>Note</u>:   The "Histograms for cell - " is done for each of the 25 cells.

		17	107	125	169	185	262	310	424	497	687	742	826	969	890	904	948	968
DT=0.33241	MAP=																	
ERROR IN LOC																		
DT=0.34043	MAP=	21	111	127	202	220	273	314	442	514	676	756	815	847	870	900	038	061
DT=0.32498	MAP=	16	116	136	229	244	316	350	473	540	699	776	825	877	897	926	953	
DT=0.31012	MAP=	22	101	127	232	249	328	354	462	532	682	778	819	850	891	923	961	
DT=0.31247	MAP=	22	105	125	193	209	286	408	465	613	712	794	820	851	877	920		
DT=0.30550	MAP=	22	118	138	200	219	261	284	389	442	587	728	775	802	932	885		
DT=0.33530	MAP=	31	100	127	207	228	275	299	383	430	561	672	723	780	812	847	888	
DT=0.40380	MAP=	28	113	135	209	231	304	326	386	432	567	671	724	747	822	860	904	
DT=0.43577	MAP=	31	127	163	214	240	311	336	404	439	555	670	745	790	833	869	916	945
DT=0.28444	MAP=	30	149	168	245	266	311	343	437	466	551	673	751	815	850	898	925	
DT=0.51370	MAP=	29	123	146	238	259	326	405	430	521	661	743	778	901	833	904	933	963
DT=0.29056	MAP=	19	116	153	244	274	317	340	401	427	602	685	733	759	786	830	854	885
DT=0.43122	MAP=	27	120	145	242	268	323	344	395	424	497	591	671	727	740	772	821	844
DT=0.27430	MAP=	24	125	161	240	249	274	293	345	373	442	549	607	652	692	716	760	781
DT=0.28063	MAP=	36	124	148	230	245	295	313	346	366	429	551	610	652	685	717	761	781
ERROR IN LOC																		
DT=0.29127	MAP=	28	99	130	215	238	282	296	334	355	406	522	603	654	681	710	756	778
DT=0.27059	MAP=	37	127	151	241	258	307	319	352	369	417	528	603	655	692	712	754	775
DT=0.48684	MAP=	25	135	175	255	277	321	337	380	393	425	514	586	631	663	696	741	762
DT=0.37829	MAP=	31	105	133	258	268	323	347	401	417	437	505	567	621	646	677	740	764
DT=0.33496	MAP=	26	100	138	207	244	327	340	391	405	433	553	607	650	690	720	781	805
DT=0.26375	MAP=	27	119	145	274	265	327	349	408	423	451	570	632	672	714	746	766	815
DT=0.37233	MAP=	23	108	144	243	306	342	424	457	564	626	674	690	751	812	845	939	
DT=0.38265	MAP=	24	106	131	182	205	268	297	375	389	425	528	613	654	682	712	765	800
DT=0.39911	MAP=	24	113	147	231	248	295	319	391	408	451	555	630	668	693	727	778	814
ERROR IN LOC																		
DT=0.36406	MAP=	24	104	133	245	263	320	341	387	398	452	563	626	673	712	747	794	830
DT=0.29700	MAP=	32	109	152	221	245	310	330	381	395	457	578	631	679	707	718	784	818
DT=0.30441	MAP=	29	120	149	220	239	280	308	360	381	430	547	606	646	670	702	747	777
DT=0.25078	MAP=	23	120	149	225	240	283	310	369	395	429	551	608	652	672	716	768	802
DT=0.31977	MAP=	27	115	131	211	233	279	305	348	367	413	541	601	656	687	716	762	793
DT=0.26965	MAP=	31	129	155	248	264	303	326	366	383	439	548	634	693	715	750	799	833
DT=0.25812	MAP=	24	107	146	230	274	315	329	368	385	443	564	642	692	721	763	810	856
DT=0.37864	MAP=	24	107	147	228	248	292	302	333	354	404	502	693	650	677	719	764	801
DT=0.32447	MAP=	27	95	124	196	212	253	266	296	315	355	478	551	608	642	671	728	760
DT=0.37647	MAP=	26	115	152	201	224	258	273	304	321	356	484	559	615	645	670	723	762
ERROR IN LOC																		
DT=0.43511	MAP=	35	137	155	240	256	307	324	358	368	394	522	596	653	685	710	767	806
DT=0.34669	MAP=	47	170	189	302	320	371	386	442	448	471	569	630	683	711	742	794	835

Note: The "Histograms for cell - " is done for each of the 25 cells

UNIVERSITY OF TORONTO

SKIMMER 1971

```
&GEO
PFI= 1.1499996    ,DISNS= 7.0000000    ,DIAMS= 0.54999995    ,THLIP= 0.99999964E-01,SLO= 0.92999995    ,SLI= 0.69999999    ,XCB1=
-1.0000000        ,XCB2= 0.5000000     ,XCB3= 1.5000000      ,XCB4= 3.0000000      ,SLI1= 0.15714276     ,SLI2= 0.78571379E-01,XCB0=    ,YCB4=
0.3631897         ,XCB1= 1.953951A     ,XCA5= 1.3631916      ,YCB1= 1.3749990      ,YCA1= 0.13749999     ,YCA2= 0.27469998     ,YCB4= 0.37469994    ,SLO3=
0.54999995        ,VCB5= 0.82499993    ,VCB6= 1.3749990      ,VCB7= 1.6477670      ,SLI1= 1.8599997      ,SLI2= 2.7499990      ,DISNI=
0.78571379E-01,SLO4= 3.11785710    ,SLO5= 0.19642842    ,SLI3= 0.392857OAE-01,SLI4= 0.19642852F-01,PADIN= 1.1785698    ,YNS=
6.0000000         ,XNI= 0.76188570     ,YNI=-0.81923199      ,XNO=-0.81101281      ,YNO= 0.73227195      ,XN5= 0.19274509
-0.98124856       ,VOL= 0.25639497E-01, 0.83392024E-01, 0.70551932F-01, 0.256752335-01, 0.31866690E-01, 0.95608950F-01, 0.77679157E-01,
0.23303765        , 0.15591383        , 0.46774197        , 0.38059275        , 0.29136091        , 0.73771054        , 0.38943589    , 0.38877684
2.9284039         , 1.7104807         , 2.9427347         , 0.20664322        , 0.8343A9A06F-01, 0.34364694        , 0.19970407    , 0.91639000
2.5216074         , 10.137089
&END
&CON
CU= 404.85156     ,SIA: 0.99999964E-01,SIB= 0.70699930E-01,CQ= 1.0000000    ,CT= 0.70699996    ,ACCOM= 1.0000000    ,PRFS=
1.0000000         ,CLWT= 1.0000000    , 1.0000000         , 1.0000000         , 1.0000000         , 1.0000000         , 1.0000000
1.0000000         , 1.0000000         , 1.0000000         , 0.25000000        , 0.25000000        , 0.62500000E-01, 0.62500000E-01,
0.62500000E-01,   0.15625000E-01, 0.15625000E-01, 0.15625000E-01, 1.0000000    , 1.0000000
1.0000000         , 1.0000000         , 0.0               , 0.0               , 61.450516         , 18.919754         , 1.0000000
51.988815         , 23.484360         , 70.453018         , 57.240784         , 171.72246         , 114.59091         , 344.67310    , 70.111633
53.675018         , 33.975586         , 17.935638         , 17.905273         , 33.717224         , 19.694229         , 33.002233    , 152.27275
61.632446         , 253.22893         , 146.48874         , 675.27612         , 1858.1342         , 7469.8906
&END
&BEG
BR=    1795.BG=    1795.A=    2.0000000    ,NST=    1000.R=    85371829O,G=    972232677
&END
&TIM
T= 12.303652      ,DT= 0.34669107    ,DTM= 0.33333331    ,DTS=M 1.0000000    ,DTSM= 1.0000000    ,TENN= 12.000000    ,DTRP=
1.0000000         ,TREP= 12.000000
&END
```

FLUX PROFILE AT ORIFICE

DISTANCE FROM AXIS   NUMBER PER UNIT TIME

```
0.02750   ****************************************        19.51538
0.05500   ***************************************         22.30328
0.08250   ***********************                         18.67900
0.11000   ***********************                         18.51970
0.13750   ***************************                     21.52AA7
0.16500   *****************                               15.A6042
0.19250   *****************                               17.47H06
0.22000   ******************                              17.28505
0.24750   ***************                                 14.51355
0.27500   ***********                                     9.63750
```

TOTAL   175.20087

NUMBER/TIME THRU ORIFICE = 137.52980. THEORETICAL FIGURE FOR NO SKIMMER INTERACTION = 17.0708?

A2-2

UNIVERSITY OF TORONTO

DISTANCE FROM AXIS		NUMBER PER UNIT TIME
0.03206	*******************************	1.16674
0.06571	**********************************	1.55566
0.09857	**	2.56684
0.13142	***	2.83352
0.16428	**	2.98168
0.19713	***	2.75776
0.22999	**	2.33349
0.26284	*************************************	1.94457
0.29570	**	2.60902
0.32855	***	2.45620
0.36141	***************************	1.44454
0.39426	********************************	7.07985
0.42712	********************	1.68011
0.45997	**************	1.38281
0.49283	*************	0.68395
0.52569	**************	0.97856
0.55854	**********	0.70712
0.59140	*****************	1.16674
0.62425	************	0.76681
0.65711	*************	0.98725
0.68996	*******	0.22766
0.72282	*******	0.43414
0.75567	********	0.44077
0.78853	******	0.29789
0.82138	**********	0.50003
0.85424	**********	0.48042
0.88709	*********	0.41877
0.91995	********	0.31820
0.95280	*******	0.34562
0.98566	******	0.27730
1.01851	*******	0.26341
1.05137	*******	0.42595
1.08423	******	0.21540
1.11708	*******	0.36570
1.14994	*****	0.29055
1.18279	*****	0.23006
1.21565	*****	0.28769
1.24850	****	0.18668
1.28136	******	0.31820
1.31421	****	0.14769
1.34707	******	0.37451
1.37992	****	0.18274
1.41278	****	0.13776
1.44563	***	0.14752
1.47849	****	0.16664
1.51134	****	0.16668
1.54470	****	0.21323
1.57706	***	0.09825
1.60991	*****	0.20448
1.64277	**********	0.46319

NUMBER/TIME IN BEAM 28.54164

TOTAL 45.6531P

A2-3

UNIVERSITY OF TORONTO

VELOCITY HISTOGRAMS FOR CELL 4

THE FIRST HISTOGRAM TRIPLE OFFERS TO MOLECULES WHICH HAVE UNDERGONE NO COLLISIONS IN THE FLOWFIELD.
THE SECOND TO MOLECULES WHICH HAVE UNDERGONE AT LEAST ONE COLLISION, AND THE THIRD GIVES THE SUM OF THE FIRST TWO.
LOWER CELL BOUNDARY VALUES APPEAR ONLY THE LEFT, TIME AVERAGED NUMBER OF MOLECULES APPEARS AT THE RIGHT OF EACH BAR.

-1.50000	*	0.70648	*	0.84152	**	0.97441
-1.20000	**	0.79846	**	1.01412	*	1.21466
-0.90000	***	2.88463	**	0.95611	**	0.67978
-0.60000	*	1.95595	***	1.39375	**	0.84047
-0.30000	*	0.62503	*********	8.81504	***********	12.34533
0.0		0.0	**********************	18.52901	*********************	14.70952
0.30000	*	0.0		0.98213	**	0.74295
0.60000	**	2.19485	*	0.04804	**	0.54111
0.90000	*********************	23.98384	*	0.49769	*	0.28260
1.20000	*	0.0	*	0.37321	**	0.87541
TOTALS		33.14963		33.14960		33.14961

-1.50000	*	0.51958	*	0.0	*	0.14307
-1.20000		0.0	*	0.26465	*	0.61572
-0.90000	**	0.67336	******	4.97926	****	7.00127
-0.60000	**	0.94652	******	5.33958	****	3.97033
-0.30000	******	3.67065	**********	11.95767	************	14.43795
0.0	**********	6.98103	*************	14.39689	**************	14.58814
0.30000	************	10.14162	*******	6.87838	*****	4.04215
0.60000	**************	12.34117	*****	3.06251	***	2.27678
0.90000	************	11.70164	**	0.96544	**	0.77821
1.20000	***	1.54171	*	0.22189	*	0.65171
TOTALS		48.51729		48.51721		48.51730

-1.50000	*	1.22606	*	0.94152	*	1.11748
-1.20000	*	0.79886	*	1.27877	*	0.87431
-0.90000	**	3.55799	**	5.93437	**	2.38055
-0.60000	**	2.90247	***	7.73733	****	4.81079
-0.30000	***	4.29568	************	20.77269	***************	27.40327
0.0	****	6.98103	********************	33.62590	*****************	29.29765
0.30000	******	10.14162	*****	6.91061	*****	9.22709
0.60000	********	14.53602	***	3.11058	***	3.15789
0.90000	****************	35.68552	**	1.36412	**	1.07082
1.20000	*	1.54171	*	0.59510	*	1.52712
TOTALS		81.66693		81.66695		81.66695

THEORETICAL NO INTERACTION TOTAL = 55.20851

MEAN VELOCITY VECTOR 0.57278 -0.00041 0.0.086   MEAN SQUARE VELOCITY VECTOR 0.64413 0.17106 0.19060
TIME AVERAGED NUMBER OF MOLECULES WHICH HAVE UNDERGONE MORE THAN ONE COLLISION IN THE FLOWFIELD 33.45020

A2-4

APPENDIX 3

No Interaction Beam Intensity

We derive here a formula for the beam intensity at points close to the skimmer.

Let D be the nozzle skimmer distance, L be the skimmer-observation point distance and b be the skimmer radius. Consider a particle in the skimmer orifice plane at position $(r \cos \theta, r \sin \theta)$ whose velocity has a probability density function proportional to

$$\exp-[(u-1)^2 + (v-r \cos \theta/D)^2 + (w-r \sin \theta/D)^2]/c^2$$

That is, the mean velocity vector points in the direction

$$(1, \frac{r \cos \theta}{D}, \frac{r \sin \theta}{D})$$

The impact position of the particle on the plane normal to the axis distance L downstream of the skimmer then has probability distribution

$$\frac{1}{L^2 d^2} \; e^{-\frac{1}{Lc}^2} [(x-\alpha r \cos \theta)^2 + (y-\alpha r \cos \theta)^2]$$

where

$$\alpha = \frac{L + D}{D}$$

If the flux through the orifice is $\emptyset$ particles per unit area then the flux through the point x,y is

$$\emptyset_L(x,y) = \emptyset \int_o^b r dr \int_o^{2\pi} d\theta \; \frac{1}{L^2 c^2 \pi} \; e^{-(\frac{1}{Lc})^2 [(x-dr \cos \theta)^2 + (y-dr \sin \theta)^2]}$$

This integral is messy in general, but if we ask only for the centreline flux ($x = y = o$) we obtain

$$\emptyset_{Lo} = \int_o^b r dr \int_o^{2\pi} d\theta \; \frac{1}{Lc^2 \pi} \; e^{-(\frac{1}{Lc})^2 \alpha^2 r^2}$$

which one can easily evaluate to obtain

$$\emptyset_{Lo} = (\frac{D}{L+D})^2 (1 - e^{-[\frac{b}{c}(\frac{1}{L} + \frac{1}{D})]^2}).$$